JEE Main 2025 (January)

Chapter-wise Qs Bank

Mathematics

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Content

Questions

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Solutions

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Q1. The product of all solutions of the equation $e^{5(\log_e x)^2+3} = x^8, x > 0$, is :

(1) $e^{8/5}$

(2) $e^{6/5}$

(3) e^2

(4) e

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Q1. The number of solutions of the equation $\left(\frac{9}{x}-\frac{9}{\sqrt{x}}+2\right)\left(\frac{2}{x}-\frac{7}{\sqrt{x}}+3\right)=0$ is:

(1) 2

(2) 3

(3) 1

(4) 4

Q2. The sum, of the squares of all the roots of the equation $x^2+|2x-3|-4=0$, is

(1) $3(3-\sqrt{2})$

(2) $6(3-\sqrt{2})$

(3) $6(2-\sqrt{2})$

(4) $3(2-\sqrt{2})$

Q3. If the set of all $\mathrm{a} \in \mathbf{R}$, for which the equation $2x^2+(a-5)x+15=3\mathrm{a}$ has no real root, is the interval (α, β), and $X=\{x \in Z: \alpha<x<\beta\}$, then $\sum_{x \in X} x^2$ is equal to :

(1) 2109

(2) 2129

(3) 2119

(4) 2139

Q4. If the equation $\mathrm{a}(\mathrm{b}-\mathrm{c})\mathrm{x}^2+\mathrm{b}(\mathrm{c}-\mathrm{a})\mathrm{x}+\mathrm{c}(\mathrm{a}-\mathrm{b})=0$ has equal roots, where $\mathrm{a}+\mathrm{c}=15$ and $\mathrm{b}=\frac{36}{5}$, then a^2+c^2 is equal to

Q5. The product of all the rational roots of the equation $\left(x^2-9x+11\right)^2-(x-4)(x-5)=3$, is equal to

(1) 14

(2) 21

(3) 28

(4) 7

Q6. Let α_θ and β_θ be the distinct roots of $2x^2+(\cos\theta)x-1=0, \theta \in(0,2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4+\beta_\theta^4$, then $16(M+m)$ equals :

(1) 24

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(2) 25

(3) 17

(4) 27

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Q1. The number of complex numbers z, satisfying $|z|=1$ and $\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1$, is :

(1) 4

(2) 8

(3) 10

(4) 6

Q2. Let $|z_1-8-2i|\leq 1$ and $|z_2-2+6i|\leq 2, z_1, z_2 \in \mathbf{C}$. Then the minimum value of $|z_1-z_2|$ is :

(1) 13

(2) 10

(3) 3

(4) 7

Q3. Let α, β be the roots of the equation $x^2-ax-b=0$ with $\operatorname{Im}(\alpha)<\operatorname{Im}(\beta)$. Let $P_n=\alpha^n-\beta^n$. If $\mathrm{P}_3=-5\sqrt{7}i, \mathrm{P}_4=-3\sqrt{7}i, \mathrm{P}_5=11\sqrt{7}i$ and $\mathrm{P}_6=45\sqrt{7}i$, then $|\alpha^4+\beta^4|$ is equal to ________ .

Q4. If $\alpha+i\beta$ and $\gamma+i\delta$ are the roots of $x^2-(3-2i)x-(2i-2)=0, i=\sqrt{-1}$, then $\alpha\gamma+\beta\delta$ is equal to :

(1) -2

(2) 6

(3) -6

(4) 2

Q5. Let z_1, z_2 and z_3 be three complex numbers on the circle $|z|=1$ with $\arg(z_1)=\frac{-\pi}{4}, \arg(z_2)=0$ and $\arg(z_3)=\frac{\pi}{4}$. If $|z_1\bar{z}_2+z_2\bar{z}_3+z_3\bar{z}_1|^2=\alpha+\beta\sqrt{2}, \alpha, \beta \in \mathbf{Z}$, then the value of $\alpha^2+\beta^2$ is :

(1) 24

(2) 29

(3) 41

(4) 31

Q6. Let O be the origin, the point A be $z_1=\sqrt{3}+2\sqrt{2}i$, the point $B(z_2)$ be such that $\sqrt{3}|z_2|=|z_1|$ and $\arg(z_2)=\arg(z_1)+\frac{\pi}{6}$. Then

(1) area of triangle ABO is $\frac{11}{\sqrt{3}}$

(2) ABO is an obtuse angled isosceles triangle

(3) area of triangle ABO is $\frac{11}{4}$

(4) ABO is a scalene triangle

Q7. Let the curve $z(1+i)+\bar{z}(1-i)=4, z \in \mathrm{C}$, divide the region $|z-3| \leq 1$ into two parts of areas α and β. Then $|\alpha-\beta|$ equals :

(1) $1+\frac{\pi}{2}$

(2) $1+\frac{\pi}{3}$

(3) $1+\frac{\pi}{6}$

(4) $1+\frac{\pi}{4}$

Q8. Let $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}, z \in C$, be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points $(0,0)$, C and $(\alpha, 0)$ is 11 square units, then α^2 equals:

(1) 50

(2) 100

(3) $\frac{81}{25}$

(4) $\frac{121}{25}$

Q9. If α and β are the roots of the equation $2z^2-3z-2\mathrm{i}=0$, where $\mathrm{i}=\sqrt{-1}$, then $16 \cdot \operatorname{Re}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right) \cdot \operatorname{Im}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)$ is equal to

(1) 441

(2) 398

(3) 312

(4) 409

Q10. Let integers a, b $\in[-3,3]$ be such that $\mathrm{a}+\mathrm{b} \neq 0$. Then the number of all possible ordered pairs (a, b), for which $\left|\frac{z-\mathrm{a}}{z+\mathrm{b}}\right|=1$ and $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1, z \in \mathrm{C}$, where ω and ω^2 are the roots of $x^2+x+1=0$, is equal to ________.

Q1. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

(1) -1080

(2) -1020

(3) -1200

(4) -120

Q2. The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88 , then $q - 2p$ is equal to ________ -.

Q3. In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to :

(1) 525

(2) 510

(3) 515

(4) 505

Q4. Consider an A. P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11^{th} term is :

(1) 90

(2) 84

(3) 122

(4) 108

Q5. The number of 3 -digit numbers, that are divisible by 2 and 3 , but not divisible by 4 and 9 , is______.

Q6. Let T_r be the r^{th} term of an A.P. If for some m, $T_m = \frac{1}{25}$, $T_{25} = \frac{1}{20}$, and $20\sum_{r=1}^{25} T_r = 13$, then $5m\sum_{r=m}^{2m} T_r$ is equal to

(1) 98

(2) 126

(3) 142

(4) 112

Q7. The interior angles of a polygon with n sides, are in an A.P. with common difference 6°. If the largest interior angle of the polygon is 219°, then n is equal to

Q8. Let $a_1, a_2, \ldots, a_{2024}$ be an Arithmetic Progression such that $a_1 + (a_5 + a_{10} + a_{15} + \ldots + a_{2020}) + a_{2024} = 2233$. Then $a_1 + a_2 + a_3 + \ldots + a_{2024}$ is equal to _______

Q9. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive terms. If $a_1 a_5 = 28$ and $a_2 + a_4 = 29$, then a_6 is equal to:

(1) 628

(2) 812

(3) 526

(4) 784

Q10. If $7 = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + \frac{1}{7^3}(5+3\alpha) +$ ________ ∞, then the value of α is :

(1) $\frac{6}{7}$

(2) 6

(3) $\frac{1}{7}$

(4) 1

Q11. If $\sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty} \sum_{r=1}^{n}\left(\frac{1}{T_r}\right)$ is equal to :

(1) 0

(2) $\frac{2}{3}$

(3) 1

(4) $\frac{1}{3}$

Q12. For positive integers n, if $4a_n = \left(n^2 + 5n + 6\right)$ and $S_n = \sum_{k=1}^{n}\left(\frac{1}{a_k}\right)$, then the value of $507 S_{2025}$ is :

(1) 540

(2) 675

(3) 1350

(4) 135

Q13. The value of $\lim_{n\to\infty}\left(\sum_{k=1}^{n} \frac{k^3+6k^2+11k+5}{(k+3)!}\right)$ is:

(1) $4/3$

(2) 2

(3) $7/3$

(4) $5/3$

Q14. Suppose that the number of terms in an A.P. is $2k, k \in N$. If the sum of all odd terms of the A.P. is 40 , the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to :

(1) 6

(2) 5

(3) 8

(4) 4

Q15. Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots$ upto n terms. If the sum of the first six terms of an A.P. with first term -p and common difference p is $\sqrt{2026\ \mathrm{S}_{2025}}$, then the absolute difference betwen 20^{th} and 15^{th} terms of the A.P. is

(1) 20

(2) 90

(3) 45

(4) 25

Q16. Let $\langle a_{\mathrm{n}} \rangle$ be a sequence such that $a_0 = 0, a_1 = \frac{1}{2}$ and $2a_{\mathrm{n}+2} = 5a_{\mathrm{n}+1} - 3a_{\mathrm{n}}, \mathrm{n} = 0, 1, 2, 3, \ldots$. Then $\sum_{\mathrm{k}=1}^{100} a_k$ is equal to

(1) $3\mathrm{a}_{99} - 100$

(2) $3\mathrm{a}_{100} - 100$

(3) $3\mathrm{a}_{99} + 100$

(4) $3\mathrm{a}_{100} + 100$

Q1. If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440^{th} position in this arrangement, is :

(1) PRNAUK

(2) PRKANU

(3) PRKAUN

(4) PRNAKU

Q2. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to :

(1) 8750

(2) 9100

(3) 8925

(4) 8575

Q3. The number of different 5 digit numbers greater than 50000 that can be formed using the digits 0 , 1, 2, 3, 4, 5, 6, 7 , such that the sum of their first and last digits should not be more than 8 , is

(1) 4608

(2) 5720

(3) 5719

(4) 4607

Q4. The number of natural numbers, between 212 and 999 , such that the sum of their digits is 15 , is

Q5. Let P be the set of seven digit numbers with sum of their digits equal to 11 . If the numbers in P are formed by using the digits 1,2 and 3 only, then the number of elements in the set P is :

(1) 173

(2) 164

(3) 158

(4) 161

Q6. The number of 6-letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice, is _______.

Q7. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is ' M ', is :

(1) 5148

(2) 6084

(3) 4356

(4) 14950

Q8. In a group of 3 girls and 4 boys, there are two boys B_1 and B_2. The number of ways, in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is :

(1) 96

(2) 144

(3) 120

(4) 72

Q9. The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is

(1) 36000

(2) 37000

(3) 34000

(4) 35000

Q10. The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is ________ -

Q1. The sum of all rational terms in the expansion of $\left(1+2^{1/2}+3^{1/2}\right)^{6}$ is equal to

Q2. For some $n \neq 10$, let the coefficients of the 5 th, 6 th and 7 th terms in the binomial expansion of $(1+\mathrm{x})^{\mathrm{n}+4}$ be in A.P. Then the largest coefficient in the expansion of $(1+\mathrm{x})^{\mathrm{n}+4}$ is:

(1) 20

(2) 10

(3) 35

(4) 70

Q3. Suppose A and B are the coefficients of 30^{th} and 12^{th} terms respectively in the binomial expansion of $(1+x)^{2\mathrm{n}-1}$. If $2\mathrm{~A}=5\mathrm{~B}$, then n is equal to :

(1) 22

(2) 20

(3) 21

(4) 19

Q4. Let α, β, γ and δ be the coefficients of x^{7}, x^{5}, x^{3} and x respectively in the expansion of $\left(x+\sqrt{x^{3}-1}\right)^{5}+\left(x-\sqrt{x^{3}-1}\right)^{5}, x>1$. If u and v satisfy the equations

$\alpha u+\beta v=18$

$\gamma u+\delta v=20$

then $u+v$ equals :

(1) 5

(2) 3

(3) 4

(4) 8

Q5. If in the expansion of $(1+x)^{\mathrm{p}}(1-x)^{\mathrm{q}}$, the coefficients of x and x^{2} are 1 and -2 , respectively, then $\mathrm{p}^{2}+\mathrm{q}^{2}$ is equal to :

(1) 18

(2) 13

(3) 8

(4) 20

Q6. Let the coefficients of three consecutive terms T_r, T_{r+1} and T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion of $(\sqrt[4]{3}+\sqrt[3]{4})^{12}$. Then p + q is equal to :

(1) 283

(2) 287

(3) 295

(4) 299

Q7. The least value of n for which the number of integral terms in the Binomial expansion of $(\sqrt[3]{7}+\sqrt[12]{11})^{n}$ is 183, is :

(1) 2184

(2) 2196

(3) 2148

(4) 2172

Q8. The remainder, when 7^{103} is divided by 23 , is equal to :

(1) 6

(2) 17

(3) 9

(4) 14

Q9. If $\sum_{r=0}^{5} \frac{{}^{11}C_{2r+1}}{2r+2} = \frac{\mathrm{m}}{\mathrm{n}}, \gcd(\mathrm{m}, \mathrm{n}) = 1$, then $\mathrm{m}-\mathrm{n}$ is equal to _______

Q10. If $\alpha = 1 + \sum_{r=1}^{6}(-3)^{r-1}\ {}^{12}\mathrm{C}_{2r-1}$, then the distance of the point $(12, \sqrt{3})$ from the line $\alpha x - \sqrt{3}y + 1 = 0$ is _________.

Q11. If $\sum_{r=1}^{30} \frac{r^2\left({}^{30}C_r\right)^2}{{}^{30}C_{r-1}} = \alpha \times 2^{29}$, then α is equal to ______

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Q1. The variance of the numbers $8, 21, 34, 47, \ldots, 320$ is

Q2. Marks obtains by all the students of class 12 are presented in a freqency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12 . If the number of students whose marks are less than 12 is 18 , then the total number of students is

(1) 52

(2) 48

(3) 44

(4) 40

Q3. For a statistical data $x_1, x_2, \ldots, x_{10}$ of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_i^2 = 371$. He later found that he had noted two values in the data incorrectly as 4 and 5 , instead of the correct values 6 and 8 , respectively. The variance of the corrected data is

(1) 9

(2) 5

(3) 7

(4) 4

Q4. Let $x_1, x_2, \ldots, x_{10}$ be ten observations such that $\sum_{i=1}^{10}(x_i - 2) = 30, \sum_{i=1}^{10}(x_i - \beta)^2 = 98, \beta > 2$, and their variance is $\frac{4}{5}$. If μ and σ^2 are respectively the mean and the variance of $2(x_1 - 1) + 4\beta, 2(x_2 - 1) + 4\beta, \ldots, 2(x_{10} - 1) + 4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to :

(1) 100

(2) 120

(3) 110

(4) 90

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Q1. Let $A=\left[a_{ij}\right]$ be 3×3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals :

(1) -1

(2) 2

(3) 1

(4) 0

Q2. Let $A=\left[a_{ij}\right]$ be a matrix of order 3×3, with $a_{ij}=(\sqrt{2})^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha+\beta\sqrt{2}, \alpha, \beta\in\mathbf{Z}$, then $\alpha+\beta$ is equal to :

(1) 280

(2) 224

(3) 210

(4) 168

Q3. For a 3×3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A|=\frac{1}{2}$ and trace $(A)=3$. If $B=\operatorname{adj}(\operatorname{adj}(2A))$, then the value of $|B|+$ trace (B) equals :

(1) 56

(2) 132

(3) 174

(4) 280

Q4. Let M denote the set of all real matrices of order 3×3 and let $\mathrm{S}=\{-3,-2,-1,1,2\}$. Let
$\mathrm{S}_1=\{\mathrm{A}=[a_{\mathrm{ij}}]\in\mathrm{M}:\mathrm{A}=\mathrm{A}^{\mathrm{T}}$ and $a_{\mathrm{ij}}\in\mathrm{S},\forall\mathrm{i},\mathrm{j}\}$
$\mathrm{S}_2=\{\mathrm{A}=[a_{\mathrm{ij}}]\in\mathrm{M}:\mathrm{A}=-\mathrm{A}^{\mathrm{T}}$ and $a_{\mathrm{ij}}\in\mathrm{S},\forall\mathrm{i},\mathrm{j}\}$
$\mathrm{S}_3=\{\mathrm{A}=[a_{\mathrm{ij}}]\in\mathrm{M}:a_{11}+a_{22}+a_{33}=0$ and $a_{\mathrm{ij}}\in\mathrm{S},\forall\mathrm{i},\mathrm{j}\}$
If $n(S_1\cup S_2\cup S_3)=125\alpha$, then α equals _______

Q5. Let $\mathrm{S}=\left\{\mathrm{m}\in\mathbf{Z}:\mathrm{A}^{\mathrm{m}^2}+\mathrm{A}^{\mathrm{m}}=3\mathrm{I}-\mathrm{A}^{-6}\right\}$, where $\mathrm{A}=\begin{bmatrix}2&-1\\1&0\end{bmatrix}$. Then $\mathrm{n}(\mathrm{S})$ is equal to ______.

Q6. Let A be a square matrix of order 3 such that $\det(A)=-2$ and $\det(3\operatorname{adj}(-6\operatorname{adj}(3A)))=2^{\mathrm{m}+\mathrm{n}}\cdot 3^{\mathrm{mn}}, \mathrm{m}>\mathrm{n}$. Then $4\mathrm{m}+2\mathrm{n}$ is equal to _______

Q7. Let A be a 3×3 matrix such that $X^T A X = O$ for all nonzero 3×1 matrices $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If

$\mathbf{A}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, \mathbf{A}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$, and $\det(\operatorname{adj}(2(\mathbf{A}+\mathbf{1}))) - 2^{\alpha}3^{\beta}5^{\gamma}, \alpha, \beta, \gamma \in N$, then $\alpha^2+\beta^2+\gamma^2$ is_____.

Q8. Let $\mathrm{A} = [\mathrm{a}_{ij}] = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$.

If A_{ij} is the cofactor of a_{ij}, $\mathrm{C}_{ij} = \sum_{\mathrm{k}=1}^{2} \mathrm{a}_{i\mathrm{k}} \mathrm{A}_{j\mathrm{k}}$, $1 \leq i, j \leq 2$, and $\mathrm{C} = [\mathrm{C}_{ij}]$, then $8|\mathrm{C}|$ is equal to :

(1) 288

(2) 222

(3) 242

(4) 262

Q9. If A, B, and $\left(\operatorname{adj}\left(\mathrm{A}^{-1}\right) + \operatorname{adj}\left(\mathrm{B}^{-1}\right)\right)$ are non-singular matrices of same order, then the inverse of $\mathrm{A}\left(\operatorname{adj}\left(\mathrm{A}^{-1}\right) + \operatorname{adj}\left(\mathrm{B}^{-1}\right)\right)^{-1}\mathrm{B}$, is equal to

(1) $\mathrm{AB}^{-1} + \mathrm{A}^{-1}\mathrm{B}$

(2) $\operatorname{adj}\left(\mathrm{B}^{-1}\right) + \operatorname{adj}\left(\mathrm{A}^{-1}\right)$

(3) $\frac{AB^{-1}}{|A|} + \frac{BA^{-1}}{|B|}$

(4) $\frac{1}{|AB|}(\operatorname{adj}(B) + \operatorname{adj}(A))$

Q10. Let $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \theta > 0$. If $\mathrm{B} = \mathrm{PAP}^{\mathrm{T}}, \mathrm{C} = \mathrm{P}^{\mathrm{T}}\mathrm{B}^{10}\mathrm{P}$ and the sum of the diagonal elements of C is $\frac{\mathrm{m}}{\mathrm{n}}$, where $\gcd(\mathrm{m}, \mathrm{n}) = 1$, then $\mathrm{m} + \mathrm{n}$ is :

(1) 127

(2) 258

(3) 65

(4) 2049

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Q1. Let M and m respectively be the maximum and the minimum values of

$$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix}, x\in \mathrm{R}$$

Then M^4-m^4 is equal to :

(1) 1280

(2) 1295

(3) 1215

(4) 1040

Q2. The system of equations

$x+y+z=6$

$x+2y+5z=9,$

$x+5y+\lambda z=\mu,$

has no solution if

(1) $\lambda=15, \mu\neq 17$

(2) $\lambda\neq 17, \mu\neq 18$

(3) $\lambda=17, \mu\neq 18$

(4) $\lambda=17, \mu=18$

Q3.

For some a, b, let $f(x)=\begin{vmatrix}\mathrm{a}+\frac{\sin x}{x} & 1 & \mathrm{b}\\ \mathrm{a} & 1+\frac{\sin x}{x} & \mathrm{b}\\ \mathrm{a} & 1 & \mathrm{b}+\frac{\sin x}{x}\end{vmatrix}, x\neq 0, \lim_{x\to 0} f(x)=\lambda+\mu\mathrm{a}+\nu\mathrm{b}$. Then

$(\lambda+\mu+\nu)^2$ is equal to :

(1) 16

(2) 25

(3) 9

(4) 36

Q4. If the system of linear equations :

$x+y+2z=6$

$2x+3y+\mathrm{a}z=\mathrm{a}+1$

$-x-3y+\mathrm{b}z=2\mathrm{b}$

where $a, b\in \mathbf{R}$, has infinitely many solutions, then $7a+3b$ is equal to :

(1) 16

(2) 12

(3) 22

(4) 9

Q5. If the system of equations
$(\lambda-1)x+(\lambda-4)y+\lambda z=5$
$\lambda x+(\lambda-1)y+(\lambda-4)z=7$
$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$
has infinitely many solutions, then $\lambda^2+\lambda$ is equal to

(1) 6

(2) 10

(3) 20

(4) 12

Q6. If the system of equations
$2x-y+z=4$
$5x+\lambda y+3z=12$
$100x-47y+\mu z=212$
has infinitely many solutions, then $\mu-2\lambda$ is equal to

(1) 57

(2) 59

(3) 55

(4) 56

Q7. If the system of equations
$x+2y-3z=2$
$2x+\lambda y+5z=5$
$14x+3y+\mu z=33$
has infinitely many solutions, then $\lambda+\mu$ is equal to :

(1) 13

(2) 10

(3) 12

(4) 11

Q8. Let $\alpha, \beta(\alpha \neq \beta)$ be the values of m , for which the equations $x+y+z=1; x+2y+4z=\mathrm{m}$ and $x+4y+10z=m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}\left(n^\alpha+n^\beta\right)$ is equal to :

(1) 3080

(2) 560

(3) 3410

(4) 440

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Q1. Let $A = [a_{ij}]$ be a square matrix of order 2 with entries either 0 or 1 . Let E be the event that A is an invertible matrix. Then the probability P(E) is :

(1) $\frac{3}{16}$

(2) $\frac{5}{8}$

(3) $\frac{3}{8}$

(4) $\frac{1}{8}$

Q2. Two number k_1 and k_2 are randomly chosen from the set of natural numbers. Then, the probability that the value of $i^{k_1} + i^{k_2}$, $(i = \sqrt{-1})$ is non-zero, equals

(1) $\frac{1}{2}$

(2) $\frac{3}{4}$

(3) $\frac{1}{4}$

(4) $\frac{2}{3}$

Q3. Let S be the set of all the words that can be formed by arranging all the letters of the word GARDEN. From the set S, one word is selected at random. The probability that the selected word will NOT have vowels in alphabetical order is :

(1) $\frac{1}{2}$

(2) $\frac{1}{4}$

(3) $\frac{2}{3}$

(4) $\frac{1}{3}$

Q4. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to :

(1) 4

(2) 14

(3) 13

(4) 11

Q5. A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64\left(\mu+\sigma^2\right)$ is :

(1) 51

(2) 64

(3) 32

(4) 48

Q6. One die has two faces marked 1 , two faces marked 2 , one face marked 3 and one face marked 4 . Another die has one face marked 1 , two faces marked 2 , two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5 , when both the dice are thrown together, is

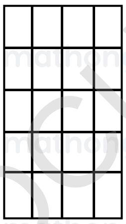
(1) $\frac{2}{3}$

(2) $\frac{1}{2}$

(3) $\frac{4}{9}$

(4) $\frac{3}{5}$

Q7. A board has 16 squares as shown in the figure:



Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is :

(1) 7/10

(2) 4/5

(3) 23/30

(4) 3/5

Q8. A and B alternately throw a pair of dice. A wins if he throws a sum of 5 before B throws a sum of 8 , and B wins if he throws a sum of 8 before A throws a sum of 5 . The probability, that A wins if A makes the first throw, is

(1) $\frac{8}{17}$

(2) $\frac{9}{19}$

(3) $\frac{9}{17}$

(4) $\frac{8}{19}$

Q9. If A and B are two events such that $P(A \cap B) = 0.1$, and $P(A \mid B)$ and $P(B \mid A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$, then the value of $\frac{\mathrm{P}(\overline{\mathrm{A}} \cup \overline{\mathrm{B}})}{\mathrm{P}(\overline{\mathrm{A}} \cap \overline{\mathrm{B}})}$ is :

(1) $\frac{4}{3}$

(2) $\frac{7}{4}$

(3) $\frac{5}{3}$

(4) $\frac{9}{4}$

Q10. Bag 1 contains 4 white balls and 5 black balls, and Bag 2 contains n white balls and 3 black balls. One ball is drawn randomly from Bag 1 and transferred to Bag 2. A ball is then drawn randomly from Bag 2. If the probability, that the ball drawn is white, is $29/45$, then n is equal to :

(1) 6

(2) 3

(3) 5

(4) 4

Q11. Bag B_1 contains 6 white and 4 blue balls, Bag B_2 contains 4 white and 6 blue balls, and Bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then the probability, that the ball is drawn from Bag B_2, is :

(1) $\frac{4}{15}$

(2) $\frac{1}{3}$

(3) $\frac{2}{5}$

(4) $\frac{2}{3}$

Q12. Let $\mathrm{A} = [\mathrm{a}_{ij}]$ be a 2×2 matrix such that $\mathrm{a}_{ij} \in \{0, 1\}$ for all i and j. Let the random variable X denote the possible values of the determinant of the matrix A. Then, the variance of X is :

(1) $\frac{3}{4}$

(2) $\frac{5}{8}$

(3) $\frac{3}{8}$

(4) $\frac{1}{4}$

Q13. Three defective oranges are accidently mixed with seven good ones and on looking at them, it is not possible to differentiate between them. Two oranges are drawn at random from the lot. If x denote the number of defective oranges, then the variance of x is

(1) 28/75

(2) 18/25

(3) $26/75$

(4) $14/25$

Q1. The number of non-empty equivalence relations on the set $\{1,2,3\}$ is :

(1) 6

(2) 5

(3) 7

(4) 4

Q2. Let $\mathrm{R}=\{(1,2),(2,3),(3,3)\}$ be a relation defined on the set $\{1,2,3,4\}$. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is:

(1) 10

(2) 7

(3) 8

(4) 9

Q3. Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by $x \mathrm{R} y$ if and only if $\sec^2 x-\tan^2 y=1$. Then R is :

(1) both reflexive and transitive but not symmetric

(2) an equivalence relation

(3) reflexive but neither symmetric not transitive

(4) both reflexive and symmetric but not transitive

Q4. Let $A=\{1,2,3, \ldots, 10\}$ and $B=\left\{\frac{m}{n}: m, n \in A, m<n \text{ and } \gcd(m, n)=1\right\}$. Then $n(B)$ is equal to :

(1) 36

(2) 31

(3) 37

(4) 29

Q5. Let $\mathrm{A}=\{(x, y) \in \mathbf{R} \times \mathbf{R}:|x+y| \geqslant 3\}$ and $\mathrm{B}=\{(x, y) \in \mathbf{R} \times \mathbf{R}:|x|+|y| \leq 3\}$.
If $\mathrm{C}=\{(x, y) \in \mathrm{A} \cap \mathrm{B}: x=0 \text{ or } y=0\}$, then $\sum_{(x, y) \in \mathrm{C}}|x+y|$ is :

(1) 15

(2) 24

(3) 18

(4) 12

Q6. Let $S=\{p_1, p_2 \ldots, p_{10}\}$ be the set of first ten prime numbers. Let $A=S \cup P$, where P is the set of all possible products of distinct elements of S. Then the number of all ordered pairs (x, y), $x \in S$, $y \in A$, such that x divides y, is ______.

Q7. Let $\mathrm{A}=\left\{x \in(0, \pi)-\left\{\frac{\pi}{2}\right\}: \log_{(2/\pi)}|\sin x|+\log_{(2/\pi)}|\cos x|=2\right\}$ and
$\mathrm{B}=\{x \geqslant 0: \sqrt{x}(\sqrt{x}-4)-3|\sqrt{x}-2|+6=0\}$. Then $\mathrm{n}(\mathrm{A} \cup \mathrm{B})$ is equal to :

(1) 4

(2) 8

(3) 6

(4) 2

Q8. Let $S=\mathbf{N} \cup\{0\}$. Define a relation R from S to $\mathbf{R}$ by :
$\mathrm{R}=\left\{(x, y): \log_{\mathrm{e}} y=x \log_{\mathrm{e}}\left(\frac{2}{5}\right), x \in \mathrm{~S}, y \in \mathbf{R}\right\}$
Then, the sum of all the elements in the range of R is equal to :

(1) $\frac{10}{9}$

(2) $\frac{3}{2}$

(3) $\frac{5}{2}$

(4) $\frac{5}{3}$

Q9. Let $A=\{1,2,3\}$. The number of relations on A, containing $(1,2)$ and $(2,3)$, which are reflexive and transitive but not symmetric, is ______ -

Q10. Let $X=\mathbf{R} \times \mathbf{R}$. Define a relation R on X as :
$(a_1, b_1) R(a_2, b_2) \Leftrightarrow b_1=b_2$
Statement I : R is an equivalence relation.
Statement II : For some $(a, b) \in X$, the set $S=\{(x, y) \in X:(x, y) R(a, b)\}$ represents a line parallel to $y=x$.
In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Q11. The relation $R = \{(x, y) : x, y \in \mathbb{Z}$ and $x + y$ is even $\}$ is:

(1) reflexive and symmetric but not transitive

(2) an equivalence relation

(3) symmetric and transitive but not reflexive

(4) reflexive and transitive but not symmetric

JEE Main 2025 January

Q1. Let $f : \mathbf{R} - \{0\} \rightarrow (-\infty, 1)$ be a polynomial of degree 2, satisfying $f(x) f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$. If $f(K) = -2K$, then the sum of squares of all possible values of K is :

(1) 7

(2) 6

(3) 1

(4) 9

Q2. Number of functions $f : \{1, 2, \ldots, 100\} \rightarrow \{0, 1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98 , is equal to ________.

Q3. Let $[x]$ denote the greatest integer less than or equal to x. Then the domain of $f(x) = \sec^{-1}(2[x] + 1)$ is :

(1) $(-\infty, -1] \cup [0, \infty)$

(2) $(-\infty, -1] \cup [1, \infty)$

(3) $(-\infty, \infty)$

(4) $(-\infty, \infty) - \{0\}$

Q4. If the domain of the function $\log_5(18x - x^2 - 77)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ), then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 195

(2) 179

(3) 186

(4) 174

Q5. Let the range of the function $f(x) = 6 + 16\cos x \cdot \cos\left(\frac{\pi}{3} - x\right) \cdot \cos\left(\frac{\pi}{3} + x\right) \cdot \sin 3x \cdot \cos 6x, x \in \mathbf{R}$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is :

(1) 11

(2) 8

(3) 10

(4) 9

Q6. Let $f(x) = \log_e x$ and $g(x) = \frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}$. Then the domain of $f \circ g$ is

(1) $[0, \infty)$

(2) $[1, \infty)$

(3) $(0, \infty)$

(4) $\mathbb{R}$

Q7. Let $\mathrm{A}=\{1,2,3,4\}$ and $\mathrm{B}=\{1,4,9,16\}$. Then the number of many-one functions $f: \mathrm{A} \rightarrow \mathrm{B}$ such that $1 \in f(\mathrm{~A})$ is equal to :

(1) 151

(2) 139

(3) 163

(4) 127

Q8. Let $f:[0,3] \rightarrow \mathrm{A}$ be defined by $f(x)=2 x^3-15 x^2+36 x+7$ and $g:[0, \infty) \rightarrow B$ be defined by $\mathrm{g}(x)=\frac{x^{2025}}{x^{2025}+1}$. If both the functions are onto and $\mathrm{S}=\{x \in \mathbf{Z}: x \in \mathrm{~A}$ or $x \in \mathrm{~B}\}$, then $\mathrm{n}(\mathrm{S})$ is equal to :

(1) 29

(2) 30

(3) 31

(4) 36

Q9. Let $f(x)=\frac{2^{x+2}+16}{2^{2 x+1}+2^{x+4}+32}$. Then the value of $8\left(f\left(\frac{1}{15}\right)+f\left(\frac{2}{15}\right)+\ldots+f\left(\frac{59}{15}\right)\right)$ is equal to

(1) 92

(2) 118

(3) 102

(4) 108

Q10. The number of real solution(s) of the equation $x^2+3 x+2=\min \{|x-3|,|x+2|\}$ is:

(1) 1

(2) 0

(3) 2

(4) 3

Q11. Let $f : R - \{0\} \rightarrow R$ be a function such that $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$.
If the $\lim_{x \to 0}\left(\frac{1}{\alpha x} + f(x)\right) = \beta; \alpha, \beta \in R$, then $\alpha + 2\beta$ is equal to

(1) 5

(2) 3

(3) 4

(4) 6

Q12. If $f(x) = \frac{2^x}{2^x + \sqrt{2}}, \mathrm{x} \in \mathbb{R}$, then $\sum_{\mathrm{k}=1}^{81} f\left(\frac{\mathrm{k}}{82}\right)$ is equal to

(1) $81\sqrt{2}$

(2) 41

(3) 82

(4) $\frac{81}{2}$

Q13. The function $f : (-\infty, \infty) \rightarrow (-\infty, 1)$, defined by $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$ is:

(1) Neither one-one nor onto

(2) Onto but not one-one

(3) Both one-one and onto

(4) One-one but not onto

Q14. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by
$f(x) = (2 + 3a)x^2 + \left(\frac{a+2}{a-1}\right)x + b, a \neq 1$. If
$f(x + \mathrm{y}) = f(x) + f(\mathrm{y}) + 1 - \frac{2}{7}x\mathrm{y}$, then the value of $28\sum_{i=1}^{5}|f(i)|$ is

(1) 545

(2) 715

(3) 735

(4) 675

JEE Main 2025 January

Q1. $\lim_{x \rightarrow 0} \operatorname{cosec} x\left(\sqrt{2 \cos^2 x+3 \cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$ is:

(1) 0

(2) $\frac{1}{\sqrt{15}}$

(3) $\frac{1}{2 \sqrt{5}}$

(4) $-\frac{1}{2 \sqrt{5}}$

Q2. If $\lim_{x \rightarrow \infty}\left(\left(\frac{\mathrm{e}}{1-\mathrm{e}}\right)\left(\frac{1}{\mathrm{e}}-\frac{x}{1+x}\right)\right)^x=\alpha$, then the value of $\frac{\log_{\mathrm{e}} \alpha}{1+\log_{\mathrm{e}} \alpha}$ equals :

(1) e^{-1}

(2) e^2

(3) e^{-2}

(4) e

Q3. If $\lim_{\mathrm{t} \rightarrow 0}\left(\int_0^1(3 x+5)^{\mathrm{t}} \mathrm{d} x\right)^{\frac{1}{t}}=\frac{\alpha}{5 \mathrm{e}}\left(\frac{8}{5}\right)^{\frac{2}{3}}$, then α is equal to ________

Q4. Let [t] be the greatest integer less than or equal to t. Then the least value of $\mathrm{p} \in \mathbf{N}$ for which $\lim_{x \rightarrow 0^{+}}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{\mathrm{p}}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots+\left[\frac{9^2}{x^2}\right]\right)\right) \geq 1$ is equal to ________.

Q5. $\lim_{x \rightarrow \infty} \frac{\left(2 x^2-3 x+5\right)(3 x-1)^{\frac{x}{2}}}{\left(3 x^2+5 x+4\right) \sqrt{(3 x+2)^x}}$ is equal to :

(1) $\frac{2}{\sqrt{3 \mathrm{e}}}$

(2) $\frac{2 \mathrm{e}}{\sqrt{3}}$

(3) $\frac{2}{3 \sqrt{\mathrm{e}}}$

(4) $\frac{2 \mathrm{e}}{3}$

Q6. Let $f(x)=\lim_{\mathrm{n} \rightarrow \infty} \sum_{\mathrm{r}=0}^{\mathrm{n}}\left(\frac{\tan \left(x / 2^{r+1}\right)+\tan^3\left(x / 2^{r+1}\right)}{1-\tan^2\left(x / 2^{r+1}\right)}\right)$. Then $\lim_{x \rightarrow 0} \frac{\mathrm{e}^x-\mathrm{e}^{f(x)}}{(x-f(x))}$ is equal to

Q1. If the function

$$f(x)=\begin{cases}\frac{2}{x}\{\sin(k_1+1)x+\sin(k_2-1)x\}, & x<0\\ 4, & x=0\\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right), & x>0\end{cases}$$

is continuous at $x=0$, then $k_1^2+k_2^2$ is equal to

(1) 20

(2) 5

(3) 8

(4) 10

Q2. Let $f(x)$ be a real differentiable function such that $f(0)=1$ and $f(x+y)=f(x)f'(y)+f'(x)f(y)$ for all $x,y\in\mathbf{R}$. Then $\sum_{n=1}^{100}\log_e f(n)$ is equal to :

(1) 2525

(2) 5220

(3) 2384

(4) 2406

Q3. Let $[x]$ denote the greatest integer function, and let m and n respectively be the numbers of the points, where the function $f(x)=[x]+|x-2|, -2<x<3$, is not continuous and not differentiable. Then $m+n$ is equal to :

(1) 6

(2) 8

(3) 9

(4) 7

Q4. Let $f(x)=\begin{cases}3x, & x<0\\ \min\{1+x+[x], x+2[x]\}, & 0\le x\le 2\\ 5, & x>2,\end{cases}$

where [.] denotes greatest integer function. If α and β are the number of points, where f is not continuous and is not differentiable, respectively, then $\alpha+\beta$ equals __________

Q5. Let the function $f(x)=\left(x^2+1\right)\left|x^2-ax+2\right|+\cos|x|$ be not differentiable at the two points $x=\alpha=2$ and $x=\beta$. Then the distance of the point (α,β) from the line $12x+5y+10=0$ is equal to :

(1) 5

(2) 4

(3) 3

(4) 2

Q6. Let the function,

$$f(x)=\begin{cases}-3ax^2-2, & x<1\\ a^2+bx, & x\geqslant 1\end{cases}$$

be differentiable for all $x \in \mathbf{R}$, where $\mathbf{a}>1, \mathbf{b} \in \mathbf{R}$. If the area of the region enclosed by $y=f(x)$ and the line $y=-20$ is $\alpha+\beta\sqrt{3}, \alpha, \beta \in Z$, then the value of $\alpha+\beta$ is ________

Q1. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is $1 \mathrm{~cm}$, the ice-cream melts at the rate of $81 \mathrm{~cm}^3/\mathrm{min}$ and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi} \mathrm{~cm}/\mathrm{min}$. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is :

(1) 196π

(2) 256π

(3) 225π

(4) 128π

Q2. Let $(2,3)$ be the largest open interval in which the function $f(x)=2\log_e(x-2)-x^2+ax+1$ is strictly increasing and (b, c) be the largest open interval, in which the function $\mathrm{g}(x)=(x-1)^3(x+2-\mathrm{a})^2$ is strictly decreasing. Then $100(a+b-c)$ is equal to :

(1) 420

(2) 360

(3) 160

(4) 280

Q3. If the set of all values of a, for which the equation $5x^3-15x-a=0$ has three distinct real roots, is the interval (α, β), then $\beta-2\alpha$ is equal to ______

Q4. The sum of all local minimum values of the function

$$f(x)=\begin{cases}1-2x, & x<-1 \\ \frac{1}{3}(7+2|x|), & -1 \leq x \leq 2 \\ \frac{11}{18}(x-4)(x-5), & x>2\end{cases}$$

is

(1) $\frac{157}{72}$

(2) $\frac{131}{72}$

(3) $\frac{171}{72}$

(4) $\frac{167}{72}$

Q1. Let $\mathrm{I}(x)=\int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$. If $\mathrm{I}(37)-\mathrm{I}(24)=\frac{1}{4}\left(\frac{1}{\mathrm{b}^{\frac{1}{13}}}-\frac{1}{\mathrm{c}^{\frac{1}{13}}}\right)$, b, c $\in$ N, then 3(b + c) is equal to

(1) 22

(2) 39

(3) 40

(4) 26

Q2. If $\int \frac{2x^2+5x+9}{\sqrt{x^2+x+1}}\,\mathrm{d}x = x\sqrt{x^2+x+1}+\alpha\sqrt{x^2+x+1}+\beta\log_e\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+\mathrm{C}$, where C is the constant of integration, then $\alpha+2\beta$ is equal to _______.

Q3. If $\int \mathrm{e}^x\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\frac{\sin^{-1}x}{(1-x^2)^{3/2}}+\frac{x}{1-x^2}\right)\mathrm{d}x=\mathrm{g}(x)+\mathrm{C}$, where C is the constant of integration, then $g\left(\frac{1}{2}\right)$ equals :

(1) $\frac{\pi}{4}\sqrt{\frac{e}{3}}$

(2) $\frac{\pi}{6}\sqrt{\frac{e}{3}}$

(3) $\frac{\pi}{4}\sqrt{\frac{e}{2}}$

(4) $\frac{\pi}{6}\sqrt{\frac{e}{2}}$

Q4. Let $\int x^3\sin x\,\mathrm{d}x=g(x)+C$, where C is the constant of integration. If $8\left(g\left(\frac{\pi}{2}\right)+g'\left(\frac{\pi}{2}\right)\right)=\alpha\pi^3+\beta\pi^2+\gamma, \alpha, \beta, \gamma \in Z$, then $\alpha+\beta-\gamma$ equals :

(1) 48

(2) 55

(3) 62

(4) 47

Q5. If $f(x)=\int\frac{1}{x^{1/4}(1+x^{1/4})}\mathrm{d}x$, $f(0)=-6$, then $f(1)$ is equal to :

(1) $4(\log_e 2-2)$

(2) $2-\log_{e^2}2$

(3) $\log_e 2+2$

(4) $4(\log_e 2+2)$

Q1. The integral $80 \int_0^{\frac{\pi}{4}} \left(\frac{\sin\theta+\cos\theta}{9+16\sin 2\theta} \right) d\theta$ is equal to :

(1) $3 \log_e 4$

(2) $4 \log_e 3$

(3) $6 \log_e 4$

(4) $2 \log_e 3$

Q2. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be a twice differentiable function such that $f(2) = 1$. If $F(x) = xf(x)$ for all $x \in \mathbf{R}$, $\int_0^2 x\, F'(x) dx = 6$ and $\int_0^2 x^2\, F''(x) dx = 40$, then $F'(2) + \int_0^2 F(x) dx$ is equal to :

(1) 11

(2) 13

(3) 15

(4) 9

Q3. If $24 \int_0^{\frac{\pi}{4}} \left(\sin\left|4x - \frac{\pi}{12}\right| + [2\sin x] \right) dx = 2\pi + \alpha$, where $[\cdot]$ denotes the greatest integer function, then α is equal to _______.

Q4. Let $f(x) = \int_0^{x^2} \frac{t^2-8t+15}{e^t} dt, x \in \mathbf{R}$. Then the numbers of local maximum and local minimum points of f, respectively, are :

(1) 2 and 3

(2) 2 and 2

(3) 3 and 2

(4) 1 and 3

Q5. Let for some function $y = f(x)$, $\int_0^x t f(t) dt = x^2 f(x), x > 0$ and $f(2) = 3$. Then $f(6)$ is equal to

(1) 1

(2) 3

(3) 6

(4) 2

Q6. Let f be a real valued continuous function defined on the positive real axis such that $g(x) = \int_0^x t f(t) dt$. If $g\left(x^3\right) = x^6 + x^7$, then value of $\sum_{r=1}^{15} f\left(r^3\right)$ is :

(1) 270

(2) 340

(3) 320

(4) 310

Q7. Let $f(x)=\int_0^t t\left(t^2-9t+20\right)dt, 1 \leq x \leq 5$. If the range of f is $[\alpha, \beta]$, then $4(\alpha+\beta)$ equals :

(1) 253

(2) 154

(3) 125

(4) 157

Q8. If $\mathrm{I}(m, n)=\int_0^1 x^{m-1}(1-x)^{n-1} dx, m, n>0$, then $I(9,14)+I(10,13)$ is

(1) $\mathrm{I}(19,27)$

(2) $\mathrm{I}(9,1)$

(3) $I(1,13)$

(4) $\mathrm{I}(9,13)$

Q9. Let for $f(x)=7\tan^8 x+7\tan^6 x-3\tan^4 x-3\tan^2 x$, $\mathrm{I}_1=\int_0^{\pi/4} f(x)\mathrm{d}x$ and $\mathrm{I}_2=\int_0^{\pi/4} x f(x)\mathrm{d}x$. Then $7\mathrm{I}_1+12\mathrm{I}_2$ is equal to :

(1) 2

(2) 1

(3) 2π

(4) π

Q10. The value of

$$\int_{e^2}^{e^4} \frac{1}{x}\left(\frac{e^{\left((\log_e x)^2+1\right)^{-1}}}{e^{\left((\log_e x)^2+1\right)^{-1}}+e^{\left((6-\log_e x)^2+1\right)^{-1}}}\right) dx \text{ is}$$

(1) 2

(2) $\log_e 2$

(3) 1

(4) e^2

Q11. If $\mathrm{I}=\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x} \mathrm{~d} x$, then $\int_0^{2 \mathrm{I}} \frac{x \sin x \cos x}{\sin^4 x+\cos^4 x} \mathrm{~d} x$ equals :

(1) $\frac{\pi^2}{12}$

(2) $\frac{\pi^2}{4}$

(3) $\frac{\pi^2}{16}$

(4) $\frac{\pi^2}{8}$

Q12. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96 x^2 \cos^2 x}{\left(1+e^x\right)} \mathrm{d} x=\pi\left(\alpha \pi^2+\beta\right), \alpha, \beta \in \mathbb{Z}$, then $(\alpha+\beta)^2$ equals

(1) 64

(2) 196

(3) 144

(4) 100

Q13. Let $f:(0, \infty) \rightarrow \mathbf{R}$ be a twice differentiable function. If for some $a \neq 0, \int_0^1 f(\lambda x) \mathrm{d} \lambda=$ $a f(x), f(1)=1$ and $f(16)=\frac{1}{8}$, then $16-f^{\prime}\left(\frac{1}{16}\right)$ is equal to _______.

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Q1. The area of the region, inside the circle $(x-2\sqrt{3})^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is :

(1) $3\pi+8$

(2) $6\pi-16$

(3) $3\pi-8$

(4) $6\pi-8$

Q2. The area of the region enclosed by the curves $y=x^2-4x+4$ and $y^2=16-8x$ is :

(1) $\frac{8}{3}$

(2) $\frac{4}{3}$

(3) 8

(4) 5

Q3. The area (in sq. units) of the region
$\{(x,y): 0 \leq \mathrm{y} \leq 2|x|+1, 0 \leq \mathrm{y} \leq x^2+1, |x| \leq 3\}$ is

(1) $\frac{80}{3}$

(2) $\frac{64}{3}$

(3) $\frac{32}{3}$

(4) $\frac{17}{3}$

Q4. The area of the region bounded by the curves $x\left(1+y^2\right)=1$ and $y^2=2x$ is:

(1) $2\left(\frac{\pi}{2}-\frac{1}{3}\right)$

(2) $\frac{\pi}{2}-\frac{1}{3}$

(3) $\frac{\pi}{4}-\frac{1}{3}$

(4) $\frac{1}{2}\left(\frac{\pi}{2}-\frac{1}{3}\right)$

Q5. The area of the region enclosed by the curves $y=\mathrm{e}^x, y=|\mathrm{e}^x-1|$ and y-axis is:

(1) $1-\log_{\mathrm{e}} 2$

(2) $\log_{\mathrm{e}} 2$

(3) $1+\log_e 2$

(4) $2 \log_e 2 - 1$

Q6. If the area of the larger portion bounded between the curves $x^2 + y^2 = 25$ and $y = |x - 1|$ is $\frac{1}{4}(b\pi + c), b, c \in N$, then $b + c$ is equal to

Q7. The area of the region $\{(x, y) : x^2 + 4x + 2 \leq y \leq |x + 2|\}$ is equal to

(1) 7

(2) 5

(3) 24/5

(4) 20/3

Q8. Let the area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ be α. If $9\alpha = \beta\pi + \gamma; \beta, \gamma$ are integers, then the value of $|\beta - \gamma|$ equals.

(1) 27

(2) 33

(3) 15

(4) 18

Q9. If the area of the region $\{(x, y) : -1 \leq x \leq 1, 0 \leq y \leq a + \mathrm{e}^{|x|} - \mathrm{e}^{-x}, \mathrm{a} > 0\}$ is $\frac{\mathrm{e}^2 + 8\mathrm{e} + 1}{\mathrm{e}}$, then the value of a is :

(1) 8

(2) 7

(3) 5

(4) 6

Q10. Let the area of the region $\{(x, y) : 2y \leq x^2 + 3, y + |x| \leq 3, y \geqslant |x - 1|\}$ be A. Then 6 A is equal to :

(1) 16

(2) 12

(3) 14

(4) 18

Q11. Consider the region $R=\left\{(x, y): x \leq y \leq 9-\frac{11}{3} x^2, x \geq 0\right\}$.

The area, of the largest rectangle of sides parallel to the coordinate axes and inscribed in R , is:

(1) $\frac{730}{119}$

(2) $\frac{625}{111}$

(3) $\frac{821}{123}$

(4) $\frac{567}{121}$

Q12. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be a twice differentiable function such that $f(x+y)=f(x) f(y)$ for all $x, y \in \mathbf{R}$. If $f^{\prime}(0)=4 \mathrm{a}$ and f satisfies $f^{\prime \prime}(x)-3 \mathrm{a} f^{\prime}(x)-f(x)=0$, $\mathrm{a}>0$, then the area of the region $\mathrm{R}=\{(x, y) \mid 0 \leq y \leq f(\mathrm{a} x), 0 \leq x \leq 2\}$ is:

(1) e^2-1

(2) e^2+1

(3) e^4+1

(4) e^4-1

Q1. Let $y = y(x)$ be the solution of the differential equation $2\cos x \frac{dy}{dx} = \sin 2x - 4y\sin x, x \in \left(0, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{3}\right) = 0$, then $y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right)$ is equal to ________ ________.

Q2. Let $x = x(y)$ be the solution of the differential equation $y = \left(x - y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right), y > 0$ and $x(1) = \frac{\pi}{2}$. Then $\cos(x(2))$ is equal to :

(1) $1 - 2(\log_e 2)^2$

(2) $1 - 2(\log_e 2)$

(3) $2(\log_e 2) - 1$

(4) $2(\log_e 2)^2 - 1$

Q3. Let $x = x(y)$ be the solution of the differential equation $y^2\,dx + \left(x - \frac{1}{y}\right)dy = 0$. If $x(1) = 1$, then $x\left(\frac{1}{2}\right)$ is :

(1) $\frac{1}{2} + e$

(2) $3 + e$

(3) $3 - e$

(4) $\frac{3}{2} + e$

Q4. If $x = f(y)$ is the solution of the differential equation
$(1 + y^2) + \left(x - 2e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0, y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
with $f(0) = 1$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(1) $e^{\pi/12}$

(2) $e^{\pi/4}$

(3) $e^{\pi/3}$

(4) $e^{\pi/6}$

Q5. Let a curve $y = f(x)$ pass through the points $(0, 5)$ and $(\log_e 2, k)$. If the curve satisfies the differential equation $2(3 + y)e^{2x}dx - (7 + e^{2x})\,dy = 0$, then k is equal to

(1) 4

(2) 32

(3) 8

(4) 16

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Q6. Let y = y(x) be the solution of the differential equation $\left(xy - 5x^2\sqrt{1+x^2}\right)dx + \left(1+x^2\right)dy = 0, y(0) = 0$. Then $y(\sqrt{3})$ is equal to

(1) $\sqrt{\frac{15}{2}}$

(2) $\frac{5\sqrt{3}}{2}$

(3) $2\sqrt{2}$

(4) $\sqrt{\frac{14}{3}}$

Q7. Let f be a differentiable function such that $2(x+2)^2 f(x) - 3(x+2)^2 = 10\int_0^x (t+2)f(t)dt, x \geq 0$. Then $f(2)$ is equal to ______.

Q8. Let $f : (0, \infty) \rightarrow \mathbf{R}$ be a function which is differentiable at all points of its domain and satisfies the condition $x^2 f'(x) = 2xf(x) + 3$, with $f(1) = 4$. Then $2f(2)$ is equal to :

(1) 39

(2) 19

(3) 29

(4) 23

Q9. If $y = y(x)$ is the solution of the differential equation,
$\sqrt{4-x^2}\frac{\mathrm{dy}}{\mathrm{dx}} = \left(\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 - y\right)\sin^{-1}\left(\frac{x}{2}\right), -2 \leq x \leq 2, y(2) = \frac{\pi^2-8}{4}$, then $y^2(0)$ is equal to

Q10. Let $y = f(x)$ be the solution of the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} + \frac{xy}{x^2-1} = \frac{x^6+4x}{\sqrt{1-x^2}}, -1 < x < 1$ such that $f(0) = 0$. If $6\int_{-1/2}^{1/2} f(x)\mathrm{d}x = 2\pi - \alpha$ then α^2 is equal to _______ .

Q11. Let $y = y(x)$ be the solution of the differential equation $\cos x\left(\log_{\mathrm{e}}(\cos x)\right)^2 \mathrm{dy} +$ $(\sin x - 3y\sin x\log_{\mathrm{e}}(\cos x))\,\mathrm{d}x = 0, x \in \left(0, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{4}\right) = \frac{-1}{\log_{\mathrm{e}} 2}$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $\frac{1}{\log_e(3) - \log_e(4)}$

(2) $\frac{2}{\log_e(3) - \log_e(4)}$

(3) $\frac{1}{\log_e(4) - \log_e(3)}$

(4) $-\frac{1}{\log_e(4)}$

Q12. If for the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx} + (\tan x)y = \frac{2+\sec x}{(1+2\sec x)^2}$, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{10}$, then $f\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{\sqrt{3}+1}{10(4+\sqrt{3})}$

(2) $\frac{5-\sqrt{3}}{2\sqrt{2}}$

(3) $\frac{9\sqrt{3}+3}{10(4+\sqrt{3})}$

(4) $\frac{4-\sqrt{2}}{14}$

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Q1. Let the area of a $\triangle PQR$ with vertices $P(5,4), Q(-2,4)$ and $R(a,b)$ be 35 square units. If its orthocenter and centroid are $O\left(2, \frac{14}{5}\right)$ and $C(c,d)$ respectively, then $c+2d$ is equal to

(1) $\frac{8}{3}$

(2) $\frac{7}{3}$

(3) 2

(4) 3

Q2. Let the triangle PQR be the image of the triangle with vertices $(1,3),(3,1)$ and $(2,4)$ in the line $x+2y=2$. If the centroid of $\triangle$PQR is the point (α,β), then $15(\alpha-\beta)$ is equal to :

(1) 19

(2) 24

(3) 21

(4) 22

Q3. Let $A(6,8), B(10\cos\alpha, -10\sin\alpha)$ and $C(-10\sin\alpha, 10\cos\alpha)$, be the vertices of a triangle. If $L(a,9)$ and $G(h,k)$ be its orthocenter and centroid respectively, then $(5a-3h+6k+100\sin 2\alpha)$ is equal to ______ -.

Q4. Let ABC be a triangle formed by the lines $7x-6y+3=0, x+2y-31=0$ and $9x-2y-19=0$. Let the point (h,k) be the image of the centroid of ΔABC in the line $3x+6y-53=0$. Then h^2+k^2+hk is equal to:

(1) 47

(2) 37

(3) 36

(4) 40

Q5. Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides $x+y=11, x+2y=16$ and $2x+3y=29$. Then the product of the smallest and the largest values of α is equal to :

(1) 44

(2) 22

(3) 33

(4) 55

Q6. Two equal sides of an isosceles triangle are along $-x+2y=4$ and $x+y=4$. If m is the slope of its third side, then the sum, of all possible distinct values of m, is :

(1) $-2\sqrt{10}$

(2) 12

(3) 6

(4) -6

Q7. Let the lines $3x-4y-\alpha=0, 8x-11y-33=0$, and $2x-3y+\lambda=0$ be concurrent. If the image of the point $(1,2)$ in the line $2x-3y+\lambda=0$ is $\left(\frac{57}{13}, \frac{-40}{13}\right)$, then $|\alpha\lambda|$ is equal to

(1) 84

(2) 113

(3) 91

(4) 101

Q8. Let ${}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}-1}=28, {}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}=56$ and ${}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}+1}=70$. Let $\mathrm{A}(4\cos t, 4\sin t), \mathrm{B}(2\sin t, -2\cos \mathrm{t})$ and $C\left(3r-n, r^{2}-n-1\right)$ be the vertices of a triangle ABC, where t is a parameter. If $(3x-1)^{2}+(3y)^{2}=\alpha$, is the locus of the centroid of triangle ABC , then α equals

(1) 6

(2) 18

(3) 8

(4) 20

Q9. Let the line $x+y=1$ meet the axes of x and y at A and B, respectively. A right angled triangle AMN is inscribed in the triangle OAB , where O is the origin and the points M and N lie on the lines OB and AB, respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and AN : NB $=\lambda:1$, then the sum of all possible value(s) of is λ :

(1) 2

(2) $\frac{5}{2}$

(3) $\frac{1}{2}$

(4) $\frac{13}{6}$

Q10. A rod of length eight units moves such that its ends A and B always lie on the lines $x-y+2=0$ and $y+2=0$, respectively. If the locus of the point P, that divides the rod AB internally in the ratio $2:1$ is $9\left(x^2+\alpha y^2+\beta xy+\gamma x+28y\right)-76=0$, then $\alpha-\beta-\gamma$ is equal to :

(1) 22

(2) 21

(3) 23

(4) 24

Q1. Let the equation of the circle, which touches x-axis at the point $(a, 0), a > 0$ and cuts off an intercept of length b on y-axis be $x^2 + y^2 - \alpha x + \beta y + \gamma = 0$. If the circle lies below x-axis, then the ordered pair $(2a, b^2)$ is equal to

(1) $(\gamma, \beta^2 - 4\alpha)$

(2) $(\alpha, \beta^2 + 4\gamma)$

(3) $(\gamma, \beta^2 + 4\alpha)$

(4) $(\alpha, \beta^2 - 4\gamma)$

Q2. Let the line $x + y = 1$ meet the circle $x^2 + y^2 = 4$ at the points A and B . If the line perpendicular to AB and passing through the mid point of the chord AB intersects the circle at C and D, then the area of the quadrilateral ADBC is equal to :

(1) $\sqrt{14}$

(2) $3\sqrt{7}$

(3) $2\sqrt{14}$

(4) $5\sqrt{7}$

Q3. Let a circle C pass through the points $(4, 2)$ and $(0, 2)$, and its centre lie on $3x + 2y + 2 = 0$. Then the length of the chord, of the circle C, whose mid-point is $(1, 2)$, is :

(1) $\sqrt{3}$

(2) $2\sqrt{2}$

(3) $2\sqrt{3}$

(4) $4\sqrt{2}$

Q4. A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point $(2, 5)$ and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to :

(1) 10

(2) 15

(3) 12

(4) 14

Q5. Let circle C be the image of $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right hand side of the centre O of C. If $B(\alpha, \beta)$, with $\beta < 4$, lies on C such that the length of the are AB is $(1/6)^{\text{th}}$ of the perimeter of C, then $\beta-\sqrt{3}\alpha$ is equal to

(1) $3+\sqrt{3}$

(2) 4

(3) $4-\sqrt{3}$

(4) 3

Q1. The focus of the parabola $y^2=4x+16$ is the centre of the circle C of radius 5 . If the values of λ, for which C passes through the point of intersection of the lines $3x-y=0$ and $x+\lambda y=4$, are λ_1 and $\lambda_2, \lambda_1<\lambda_2$, then $12\lambda_1+29\lambda_2$ is equal to ________

Q2. If the equation of the parabola with vertex V $\left(\frac{3}{2}, 3\right)$ and the directrix $x+2y=0$ is $\alpha x^2+\beta y^2-\gamma xy-30x-60y+225=0$, then $\alpha+\beta+\gamma$ is equal to :

(1) 7

(2) 9

(3) 8

(4) 6

Q3. Two parabolas have the same focus $(4,3)$ and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersects at the points A and B, then $(AB)^2$ is equal to :

(1) 392

(2) 384

(3) 192

(4) 96

Q4. Let the parabola $y=x^2+\mathrm{p}x-3$, meet the coordinate axes at the points P, Q and R . If the circle C with centre at $(-1,-1)$ passes through the points P, Q and R, then the area of $\triangle PQR$ is :

(1) 7

(2) 4

(3) 6

(4) 5

Q5. Let the shortest distance from $(\mathrm{a}, 0), \mathrm{a}>0$, to the parabola $y^2=4x$ be 4 . Then the equation of the circle passing through the point $(a, 0)$ and the focus of the parabola, and having its centre on the axis of the parabola is :

(1) $x^2+y^2-10x+9=0$

(2) $x^2+y^2-6x+5=0$

(3) $x^2+y^2-4x+3=0$

(4) $x^2+y^2-8x+7=0$

Q6. Let A and B be the two points of intersection of the line $y+5=0$ and the mirror image of the parabola $y^2=4x$ with respect to the line $x+y+4=0$. If d denotes the distance between A and B , and a denotes the area of $\triangle SAB$, where S is the focus of the parabola $y^2=4x$, then the value of $(a+d)$ is ________ -

Q7. If the line $3x-2y+12=0$ intersects the parabola $4y=3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

(1) $\tan^{-1}\left(\frac{4}{5}\right)$

(2) $\tan^{-1}\left(\frac{9}{7}\right)$

(3) $\tan^{-1}\left(\frac{11}{9}\right)$

(4) $\frac{\pi}{2}-\tan^{-1}\left(\frac{3}{2}\right)$

Q8. Let $ABCD$ be a trapezium whose vertices lie on the parabola $y^2=4x$. Let the sides AD and BC of the trapezium be parallel to y -axis. If the diagonal AC is of length $\frac{25}{4}$ and it passes through the point $(1,0)$, then the area of $ABCD$ is

(1) $\frac{75}{4}$

(2) $\frac{25}{2}$

(3) $\frac{125}{8}$

(4) $\frac{75}{8}$

Q9. Let $\mathrm{P}(4,4\sqrt{3})$ be a point on the parabola $y^2=4\mathrm{a}x$ and PQ be a focal chord of the parabola. If M and N are the foot of perpendiculars drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to :

(1) $17\sqrt{3}$

(2) $\frac{263\sqrt{3}}{8}$

(3) $\frac{34\sqrt{3}}{3}$

(4) $\frac{343\sqrt{3}}{8}$

Q10. Let $y^2=12x$ be the parabola and S be its focus. Let PQ be a focal chord of the parabola such that $(\mathrm{SP})(\mathrm{SQ})=\frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2+64y^2-\alpha x-64\sqrt{3}y=\beta$, then $\beta-\alpha$ is equal to ________.

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Q1. Let the product of the focal distances of the point $\left(\sqrt{3}, \frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (\mathrm{a}>\mathrm{b})$, be $\frac{7}{4}$. Then the absolute difference of the eccentricities of two such ellipses is

(1) $\frac{1-\sqrt{3}}{\sqrt{2}}$

(2) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

(3) $\frac{3-2\sqrt{2}}{3\sqrt{2}}$

(4) $\frac{1-2\sqrt{2}}{\sqrt{3}}$

Q2. Let the ellipse $\mathrm{E}_1: \frac{x^2}{\mathrm{a}^2}+\frac{y^2}{\mathrm{b}^2}=1, \mathrm{a}>\mathrm{b}$ and $\mathrm{E}_2: \frac{x^2}{\mathrm{A}^2}+\frac{y^2}{\mathrm{B}^2}=1,\ \mathrm{A}<\mathrm{B}$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A, B, C and D, then the area of the quadrilateral $ABCD$ equals :

(1) $\frac{12\sqrt{6}}{5}$

(2) $6\sqrt{6}$

(3) $\frac{18\sqrt{6}}{5}$

(4) $\frac{24\sqrt{6}}{5}$

Q3. The length of the chord of the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$, whose mid-point is $\left(1, \frac{1}{2}\right)$, is :

(1) $\frac{5}{3}\sqrt{15}$

(2) $\frac{1}{3}\sqrt{15}$

(3) $\frac{2}{3}\sqrt{15}$

(4) $\sqrt{15}$

Q4. The equation of the chord, of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is $(3,1)$ is :

(1) $48x+25y=169$

(2) $5x+16y=31$

(3) $25x+101y=176$

(4) $4x+122y=134$

Q5. If the midpoint of a chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is $(\sqrt{2}, 4/3)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is :

(1) 20

(2) 22

(3) 18

(4) 26

Q6. If $\alpha x+\beta y=109$ is the equation of the chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, whose mid point is $\left(\frac{5}{2}, \frac{1}{2}\right)$, then $\alpha+\beta$ is equal to :

(1) 58

(2) 46

(3) 37

(4) 72

Q7. Let $\mathrm{E}_1 : \frac{x^2}{9}+\frac{y^2}{4}=1$ be an ellipse. Ellipses E_1 's are constructed such that their centres and eccentricities are same as that of E_1, and the length of minor axis of E_i is the length of major axis of $E_{i+1}(i \geq 1)$. If A_i is the area of the ellipse E_i, then $\frac{5}{\pi}\left(\sum_{i=1}^{\infty} A_i\right)$, is equal to

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Q1. Let the foci of a hyperbola be $(1,14)$ and $(1,-12)$. If it passes through the point $(1,6)$, then the length of its latus-rectum is :

(1) $\frac{24}{5}$

(2) $\frac{25}{6}$

(3) $\frac{144}{5}$

(4) $\frac{288}{5}$

Q2. Let $\mathrm{H}_1 : \frac{x^2}{\mathrm{a}^2} - \frac{y^2}{\mathrm{b}^2} = 1$ and $\mathrm{H}_2 : -\frac{x^2}{\mathrm{A}^2} + \frac{y^2}{\mathrm{B}^2} = 1$ be two hyperbolas having length of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their ecentricities be $e_1 = \sqrt{\frac{5}{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25\mathrm{e}_2^2$ is equal to ________.

Q3. If A and B are the points of intersection of the circle $x^2+y^2-8x=0$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and a point P moves on the line $2x-3y+4=0$, then the centroid of $\triangle \mathrm{PAB}$ lies on the line :

(1) $x+9y=36$

(2) $4x-9y=12$

(3) $6x-9y=20$

(4) $9x-9y=32$

Q4. Let $\mathrm{E} : \frac{x^2}{\mathrm{a}^2} + \frac{y^2}{\mathrm{b}^2} = 1, \mathrm{a} > \mathrm{b}$ and $\mathrm{H} : \frac{x^2}{\mathrm{A}^2} - \frac{y^2}{\mathrm{B}^2} = 1$. Let the distance between the foci of E and the foci of H be $2\sqrt{3}$. If $a-A=2$, and the ratio of the eccentricities of E and H is $\frac{1}{3}$, then the sum of the lengths of their latus rectums is equal to:

(1) 10

(2) 9

(3) 8

(4) 7

Q5. Let the circle C touch the line $x-y+1=0$, have the centre on the positive x -axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x+2y=1$. Let H be the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2+3\beta^2$ is equal to _______

Q1. If $\sin x+\sin^2 x=1, x\in\left(0,\frac{\pi}{2}\right)$, then $\left(\cos^{12} x+\tan^{12} x\right)+3\left(\cos^{10} x+\tan^{10} x+\cos^8 x+\tan^8 x\right)+\left(\cos^6 x+\tan^6 x\right)$ is equal to :

(1) 4

(2) 1

(3) 3

(4) 2

Q2. The value of $(\sin 70^\circ)(\cot 10^\circ \cot 70^\circ-1)$ is

(1) $2/3$

(2) 1

(3) 0

(4) $3/2$

Q3. If $\frac{\pi}{2}\leq x\leq\frac{3\pi}{4}$, then $\cos^{-1}\left(\frac{12}{13}\cos x+\frac{5}{13}\sin x\right)$ is equal to

(1) $x-\tan^{-1}\frac{4}{3}$

(2) $x+\tan^{-1}\frac{4}{5}$

(3) $x-\tan^{-1}\frac{5}{12}$

(4) $x+\tan^{-1}\frac{5}{12}$

Q4. If $\sum_{r=1}^{13}\left\{\frac{1}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}\right\}=a\sqrt{3}+b, a, b\in\mathbf{Z}$, then a^2+b^2 is equal to :

(1) 10

(2) 4

(3) 2

(4) 8

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Q1. The sum of all values of $\theta \in [0, 2\pi]$ satisfying $2\sin^2\theta = \cos 2\theta$ and $2\cos^2\theta = 3\sin\theta$ is

(1) 4π

(2) $\frac{5\pi}{6}$

(3) π

(4) $\frac{\pi}{2}$

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Q1. If for some $\alpha, \beta; \alpha \leq \beta, \alpha + \beta - 8$ and $\sec^2\left(\tan^{-1}\alpha\right) + \operatorname{cosec}^2\left(\cot^{-1}\beta\right) - 36$, then $\alpha^2 + \beta$ is______.

Q2. $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$ is equal to:

(1) 1

(2) 0

(3) $\frac{32}{65}$

(4) $\frac{33}{65}$

Q3. Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of $16\left(\left(\sec^{-1}x\right)^2 + \left(\operatorname{cosec}^{-1}x\right)^2\right)$ is :

(1) $24\pi^2$

(2) $22\pi^2$

(3) $31\pi^2$

(4) $18\pi^2$

Q4. If $\alpha > \beta > \gamma > 0$, then the expression $\cot^{-1}\left\{\beta + \frac{(1+\beta^2)}{(\alpha-\beta)}\right\} + \cot^{-1}\left\{\gamma + \frac{(1+\gamma^2)}{(\beta-\gamma)}\right\} + \cot^{-1}\left\{\alpha + \frac{(1+\alpha^2)}{(\gamma-\alpha)}\right\}$ is equal to :

(1) π

(2) 0

(3) $\frac{\pi}{2} - (\alpha + \beta + \gamma)$

(4) 3π

Q5. Let $\mathrm{S} = \left\{x : \cos^{-1}x = \pi + \sin^{-1}x + \sin^{-1}(2x+1)\right\}$. Then $\sum_{x \in \mathrm{S}}(2x-1)^2$ is equal to ______.

Q1. Let $\overrightarrow{\mathrm{a}}=3\hat{i}-\hat{j}+2\hat{k}$, $\overrightarrow{\mathrm{b}}=\overrightarrow{\mathrm{a}}\times(\hat{i}-2\hat{k})$ and $\overrightarrow{\mathrm{c}}=\overrightarrow{\mathrm{b}}\times\hat{k}$. Then the projection of $\overrightarrow{\mathrm{c}}-2\hat{j}$ on $\vec{a}$ is :

(1) $2\sqrt{14}$

(2) $\sqrt{14}$

(3) $3\sqrt{7}$

(4) $2\sqrt{7}$

Q2. If the components of $\overrightarrow{\mathrm{a}}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ along and perpendicular to $\overrightarrow{\mathrm{b}}=3\hat{i}+\hat{j}-\hat{k}$ respectively, are $\frac{16}{11}(3\hat{i}+\hat{j}-\hat{k})$ and $\frac{1}{11}(-4\hat{i}-5\hat{j}-17\hat{k})$, then $\alpha^2+\beta^2+\gamma^2$ is equal to :

(1) 26

(2) 18

(3) 23

(4) 16

Q3. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{3}$. If $\lambda\vec{a}+2\vec{b}$ and $3\vec{a}-\lambda\vec{b}$ are perpendicular to each other, then the number of values of λ in $[-1,3]$ is :

(1) 2

(2) 1

(3) 0

(4) 3

Q4. Let the arc AC of a circle subtend a right angle at the centre O. If the point B on the arc AC, divides the arc AC such that $\frac{\text{length of arc } AB}{\text{length of arc } BC}=\frac{1}{5}$, and $\overrightarrow{OC}=\alpha\overrightarrow{OA}+\beta\overrightarrow{OB}$, then $\alpha+\sqrt{2}(\sqrt{3}-1)\beta$ is equal to

(1) $2\sqrt{3}$

(2) $2-\sqrt{3}$

(3) $5\sqrt{3}$

(4) $2+\sqrt{3}$

Q5. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=3\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{C}$ is perpendicular to $\vec{b}$ and $\vec{a}\cdot\vec{c}=5$, then $|\vec{c}|$ is equal to

(1) $\sqrt{\frac{11}{6}}$

(2) $\frac{1}{3\sqrt{2}}$

(3) 16

(4) 18

Q6. Let A, B, C be three points in xy-plane, whose position vector are given by $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\mathrm{a}\hat{i}+(1-\mathrm{a})\hat{j}$ respectively with respect to the origin O . If the distance of the point C from the line bisecting the angle between the vectors $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{OB}}$ is $\frac{9}{\sqrt{2}}$, then the sum of all the possible values of a is :

(1) 2

(2) $9/2$

(3) 1

(4) 0

Q7. Let $\vec{c}$ be the projection vector of $\vec{b}=\lambda\hat{i}+4\hat{k}, \lambda>0$, on the vector $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$. If $|\vec{a}+\vec{c}|=7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is ________

Q8. Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r}, -5\vec{p}+\vec{q}+2\vec{r}$ and $2\overrightarrow{\mathrm{p}}-\overrightarrow{\mathrm{q}}+2\overrightarrow{\mathrm{r}}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ respectively, then $\alpha+2\beta+5\gamma$ is equal to :

(1) 3

(2) 4

(3) 1

(4) 6

Q9. Let $\vec{a}=\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}, \overrightarrow{\mathrm{b}}=2\hat{\mathrm{i}}+2\hat{\mathrm{j}}+\hat{\mathrm{k}}$ and $\overrightarrow{\mathrm{d}}=\vec{a}\times\overrightarrow{\mathrm{b}}$. If $\overrightarrow{\mathrm{c}}$ is a vector such that $\vec{a}\cdot\overrightarrow{\mathrm{c}}=|\overrightarrow{\mathrm{c}}|, |\overrightarrow{\mathrm{c}}-2\vec{a}|^2=8$ and the angle between $\overrightarrow{\mathrm{d}}$ and $\overrightarrow{\mathrm{c}}$ is $\frac{\pi}{4}$, then $|10-3\overrightarrow{\mathrm{b}}\cdot\overrightarrow{\mathrm{c}}|+|\overrightarrow{\mathrm{d}}\times\overrightarrow{\mathrm{c}}|^2$ is equal to

Q10. Let $\overrightarrow{\mathrm{a}}=2\hat{i}-\hat{j}+3\hat{k}, \overrightarrow{\mathrm{b}}=3\hat{i}-5\hat{j}+\hat{k}$ and $\overrightarrow{\mathrm{c}}$ be a vector such that $\overrightarrow{\mathrm{a}}\times\overrightarrow{\mathrm{c}}=\overrightarrow{\mathrm{c}}\times\overrightarrow{\mathrm{b}}$ and $(\vec{a}+\vec{c})\cdot(\vec{b}+\vec{c})=168$. Then the maximum value of $|\vec{c}|^2$ is :

(1) 462

(2) 77

(3) 154

(4) 308

Q11. Let $\hat{a}$ be a unit vector perpendicular to the vectors $\overrightarrow{\mathrm{b}} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\overrightarrow{\mathrm{c}} = 2\hat{i} + 3\hat{j} - \hat{k}$, and makes an angle of $\cos^{-1}\left(-\frac{1}{3}\right)$ with the vector $\hat{i} + \hat{j} + \hat{k}$. If â makes an angle of $\frac{\pi}{3}$ with the vector $\hat{i} + \alpha\hat{j} + \hat{k}$, then the value of α is :

(1) $\sqrt{6}$

(2) $-\sqrt{6}$

(3) $-\sqrt{3}$

(4) $\sqrt{3}$

Q12. Let the position vectors of the vertices A, B and C of a tetrahedron $ABCD$ be $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is

(1) $\frac{1}{12}(7\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{k})$

(2) $\frac{1}{2}(\hat{i} + 4\hat{j} + 7\hat{k})$

(3) $\frac{1}{6}(12\hat{i} + 12\hat{j} + \hat{k})$

(4) $\frac{1}{6}(7\hat{\mathrm{i}} + 12\hat{\mathrm{j}} + \hat{\mathrm{k}})$

Q1. Let $L_1 : \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}, \alpha \in \mathbf{R}$, be two lines, which intersect at the point B. If P is the foot of perpendicular from the point $A(1,1,-1)$ on L_2, then the value of $26\alpha(\mathrm{PB})^2$ is ________

Q2. The distance of the line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ from the point $(1,4,0)$ along the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is :

(1) $\sqrt{17}$

(2) $\sqrt{15}$

(3) $\sqrt{14}$

(4) $\sqrt{13}$

Q3. If the square of the shortest distance between the lines $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-3}$ and $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then $m+n$ is equal to :

(1) 21

(2) 9

(3) 14

(4) 6

Q4. If the image of the point $(4,4,3)$ in the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{3}$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) 9

(2) 12

(3) 7

(4) 8

Q5. Let $L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$ and $L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$ be two lines.

Let L_3 be a line passing through the point (α, β, γ) and be perpendicular to both L_1 and L_2. If L_3 intersects L_1, then $|5\alpha - 11\beta - 8\gamma|$ equals :

(1) 20

(2) 18

(3) 25

(4) 16

Q6. Let a straight line L pass through the point $P(2,-1,3)$ and be perpendicular to the lines $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-2}$ and $\frac{x-3}{1}=\frac{y-2}{3}=\frac{z+2}{4}$. If the line L intersects the yz-plane at the point Q, then the distance between the points P and Q is :

(1) $\sqrt{10}$

(2) $2\sqrt{3}$

(3) 2

(4) 3

Q7. Let $\mathrm{A}(x,y,z)$ be a point in xy-plane, which is equidistant from three points $(0,3,2),(2,0,3)$ and $(0,0,1)$. Let $\mathrm{B}=(1,4,-1)$ and $\mathrm{C}=(2,0,-2)$. Then among the statements
(S1) : $\triangle$ABC is an isosceles right angled triangle, and
(S2) : the area of $\triangle$ABC is $\frac{9\sqrt{2}}{2}$,

(1) both are true

(2) only (S2) is true

(3) only (S1) is true

(4) both are false

Q8. Let $\mathrm{L}_1:\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\mathrm{L}_2:\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 ?

(1) $\left(\frac{14}{3},-3,\frac{22}{3}\right)$

(2) $\left(-\frac{5}{3},-7,1\right)$

(3) $\left(2,3,\frac{1}{3}\right)$

(4) $\left(\frac{8}{3},-1,\frac{1}{3}\right)$

Q9. Let a line pass through two distinct points $P(-2,-1,3)$ and Q, and be parallel to the vector $3\hat{i}+2\hat{j}+2\hat{k}$. If the distance of the point Q from the point $\mathrm{R}(1,3,3)$ is 5 , then the square of the area of $\triangle PQR$ is equal to :

(1) 148

(2) 136

(3) 144

(4) 140

Q10. The perpendicular distance, of the line $\frac{x-1}{2}=\frac{y+2}{-1}=\frac{z+3}{2}$ from the point $\mathrm{P}(2,-10,1)$, is :

(1) 6

(2) $5\sqrt{2}$

(3) $4\sqrt{3}$

(4) $3\sqrt{5}$

Q11. Let the distance between two parallel lines be 5 units and a point P lie between the lines at a unit distance from one of them. An equilateral triangle PQR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to _______ -.

Q12. Let P be the foot of the perpendicular from the point $Q(10,-3,-1)$ on the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}$. Then the area of the right angled triangle PQR, where R is the point $(3,-2,1)$, is

(1) $9\sqrt{15}$

(2) $\sqrt{30}$

(3) $8\sqrt{15}$

(4) $3\sqrt{30}$

Q13. Let in a $\triangle ABC$, the length of the side AC be 6 , the vertex B be $(1,2,3)$ and the vertices A, C lie on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Then the area (in sq. units) of $\triangle \mathrm{ABC}$ is:

(1) 17

(2) 21

(3) 56

(4) 42

Q14. Let the line passing through the points $(-1,2,1)$ and parallel to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ intersect the line $\frac{x+2}{3}=\frac{y-3}{2}=\frac{z-4}{1}$ at the point P. Then the distance of P from the point $Q(4,-5,1)$ is

(1) 5

(2) $5\sqrt{5}$

(3) $5\sqrt{6}$

(4) 10

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Q15. Let P be the image of the point $Q(7, -2, 5)$ in the line $L : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ and $R(5, p, q)$ be a point on L. Then the square of the area of $\triangle PQR$ is ________.

Q16. The square of the distance of the point $\left(\frac{15}{7}, \frac{32}{7}, 7\right)$ from the line $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ in the direction of the vector $\hat{i} + 4\hat{j} + 7\hat{k}$ is :

(1) 54

(2) 44

(3) 41

(4) 66

Q17. Let $\overrightarrow{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\overrightarrow{b} = 2\hat{i} + 7\hat{j} + 3\hat{k}$. Let $L_1 : \overrightarrow{r} = (-\hat{i} + 2\hat{j} + \hat{k}) + \lambda\overrightarrow{a}, \lambda \in \mathbf{R}$ and $L_2 : \overrightarrow{r} = (\hat{j} + \hat{k}) + \mu\overrightarrow{b}, \mu \in \mathbf{R}$ be two lines. If the line L_3 passes through the point of intersection of L_1 and L_2, and is parallel to $\vec{a} + \vec{b}$, then L_3 passes through the point :

(1) $(5, 17, 4)$

(2) $(2, 8, 5)$

(3) $(8, 26, 12)$

(4) $(-1, -1, 1)$

Q18. Let the point A divide the line segment joining the points $P(-1, -1, 2)$ and $Q(5, 5, 10)$ internally in the ratio $r : 1 (r > 0)$. If O is the origin and $(\overrightarrow{OQ} \cdot \overrightarrow{OA}) - \frac{1}{5}|\overrightarrow{OP} \times \overrightarrow{OA}|^2 = 10$, then the value of r is :

(1) $\sqrt{7}$

(2) 14

(3) 3

(4) 7

Q19. Let P be the foot of the perpendicular from the point $(1, 2, 2)$ on the line $L : \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$. Let the line $\vec{r} = (-\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}), \lambda \in \mathbf{R}$, intersect the line L at Q . Then $2(PQ)^2$ is equal to :

(1) 25

(2) 19

(3) 29

(4) 27

Basic of Mathematics

1. (1)

Quadratic Equation

1. (4) **2.** (3) **3.** (4) **4.** (117) **5.** (1) **6.** (2)

Complex Number

1. (2) **2.** (4) **3.** (31) **4.** (4) **5.** (2) **6.** (2) **7.** (1) **8.** (2)
9. (1) **10.** (10)

Sequences and Series

1. (1) **2.** (474) **3.** (3) **4.** (1) **5.** (125) **6.** (2) **7.** (20) **8.** (11132)
9. (4) **10.** (2) **11.** (2) **12.** (2) **13.** (4) **14.** (2) **15.** (4) **16.** (2)

Permutation Combination

1. (3) **2.** (3) **3.** (4) **4.** (64) **5.** (4) **6.** (1405) **7.** (1) **8.** (2)
9. (1) **10.** (17280)

Binomial Theorem

1. (612) **2.** (3) **3.** (3) **4.** (1) **5.** (2) **6.** (1) **7.** (1) **8.** (4)
9. (2035) **10.** (5) **11.** (465)

Statistics

1. (8788) **2.** (3) **3.** (3) **4.** (1)

Matrices

1. (1) **2.** (2) **3.** (4) **4.** (1613) **5.** (2) **6.** (34) **7.** (44) **8.** (3)
9. (4) **10.** (3)

Determinants

1. (1) **2.** (3) **3.** (1) **4.** (1) **5.** (4) **6.** (1) **7.** (3) **8.** (4)

Probability

1. (3) **2.** (2) **3.** (1) **4.** (2) **5.** (4) **6.** (2) **7.** (2) **8.** (2)
9. (4) **10.** (1) **11.** (1) **12.** (3) **13.** (1)

Sets and Relations

1. (2) **2.** (2) **3.** (2) **4.** (2) **5.** (4) **6.** (5120) **7.** (2) **8.** (4)
9. (3) **10.** (2) **11.** (2)

Functions

1. (2) **2.** (392) **3.** (3) **4.** (3) **5.** (1) **6.** (4) **7.** (1) **8.** (2)
9. (2) **10.** (3) **11.** (3) **12.** (4) **13.** (4) **14.** (4)

Limits

1. (4) **2.** (4) **3.** (64) **4.** (24) **5.** (3) **6.** (1)

Continuity and Differentiability

1. (4) **2.** (1) **3.** (2) **4.** (5) **5.** (3) **6.** (34)

Application of Derivatives

1. (2) **2.** (2) **3.** (30) **4.** (1)

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Indefinite Integration

1. (2) **2.** (16) **3.** (2) **4.** (2) **5.** (1)

Definite Integration

1. (2) **2.** (1) **3.** (12) **4.** (1) **5.** (1) **6.** (4) **7.** (4) **8.** (4)
9. (2) **10.** (3) **11.** (3) **12.** (4) **13.** (112)

Area Under Curves

1. (2) **2.** (1) **3.** (2) **4.** (2) **5.** (1) **6.** (77) **7.** (4) **8.** (2)
9. (3) **10.** (3) **11.** (4) **12.** (1)

Differential Equations

1. (1) **2.** (4) **3.** (3) **4.** (4) **5.** (3) **6.** (2) **7.** (19) **8.** (1)
9. (4) **10.** (27) **11.** (1) **12.** (4)

Straight Lines

1. (4) **2.** (4) **3.** (145) **4.** (2) **5.** (3) **6.** (3) **7.** (3) **8.** (4)
9. (1) **10.** (3)

Circle

1. (4) **2.** (3) **3.** (3) **4.** (2) **5.** (2)

Parabola

1. (15) **2.** (2) **3.** (3) **4.** (3) **5.** (2) **6.** (14) **7.** (2) **8.** (1)
9. (4) **10.** (1328)

Ellipse

1. (2) **2.** (4) **3.** (3) **4.** (1) **5.** (2) **6.** (1) **7.** (54)

Hyperbola

1. (4) **2.** (55) **3.** (3) **4.** (3) **5.** (19)

Trigonometric Ratios & Identities

1. (4) **2.** (2) **3.** (3) **4.** (4)

Trigonometric Equations

1. (3)

Inverse Trigonometric Functions

1. (14) **2.** (2) **3.** (2) **4.** (1) **5.** (5)

Vector Algebra

1. (1) **2.** (1) **3.** (3) **4.** (2) **5.** (1) **6.** (3) **7.** (16) **8.** (1)
9. (6) **10.** (4) **11.** (2) **12.** (4)

Three Dimensional Geometry

1. (216) **2.** (3) **3.** (2) **4.** (1) **5.** (3) **6.** (4) **7.** (3) **8.** (1)
9. (2) **10.** (4) **11.** (28) **12.** (4) **13.** (2) **14.** (2) **15.** (957) **16.** (4)
17. (3) **18.** (4) **19.** (4)

Q1. $e^{5(\ln x)^2+3} = x^8$

(1) $\Rightarrow \ell n e^{5(\ln x)^2+3} = \ell n n^8$

$\Rightarrow 5(\ln x)^2 + 3 = 8\ell nx$

$(\ell nx = t)$

$\Rightarrow 5t^2 - 8t + 3 = 0$

$t_1 + t_2 = \frac{8}{5}$

$\ln x_1 x_2 = \frac{8}{5}$

$x_1 X_2 = e^{8/5}$

Q1. Consider $\frac{1}{\sqrt{x}} = \alpha \quad x > 0$

(4) $\{9\alpha^2 - 9\alpha + 2\}\{2\alpha^2 - 7\alpha + 3\} = 0$

$(3\alpha - 2)(3\alpha - 1)(\alpha - 3)(2\alpha - 1) = 0$

$\alpha = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 3$

$x = 9, 4, \frac{9}{4}, \frac{1}{9}$

So, no. of solutions $= 4$

Q2. For $x \geq \frac{3}{2}$

(3) $x^2 + 2x - 3 - 4 = 0$

$x^2 + 2x - 7 = 0$

$x = \frac{-2 \pm \sqrt{4+28}}{2} = -1 \pm 2\sqrt{2}$

Only $2\sqrt{2} - 1$ is acceptable root

For $x < \frac{3}{2}$

$x^2 - 2x + 3 - 4 = 0$

$x^2 - 2x - 1 = 0$

$x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

Only $1 - \sqrt{2}$ is acceptable

Sum of the square $= (1 - \sqrt{2})^2 + (2\sqrt{2} - 1)^2$

$= 6(2 - \sqrt{2})$

Q3. $(a-5)^2 - 8(15 - 3a) < 0$

(4) $a^2 + 14a + 25 - 120 < 0$

$a^2 + 14a - 95 < 0$

$(a + 19)(a - 5) < 0$

$a \in (-19, 5)$

$\therefore -19 < x < 5$

$\therefore \sum_{x \in X} x^2 = \left(1^2 + 2^2 + \ldots. + 4^2\right) + \left(1^2 + 2^2 + \ldots + 18^2\right)$

$= \frac{4 \times 5 \times 9}{6} + \frac{18 \times 19 \times 37}{6}$

$= 30 + 2109$

$= 2139$

Q4. $a(b-c)x^2+b(c-a)x+c(a-b)=0$

(117) $x=1$ is root $\therefore$ other root is 1

$\alpha+\beta=-\frac{b(c-a)}{a(b-c)}=2$

$\Rightarrow -\mathrm{bc}+\mathrm{ab}=2\mathrm{ab}-2\mathrm{ac}$

$\Rightarrow 2\mathrm{ac}=\mathrm{ab}+\mathrm{bc}$

$\Rightarrow 2\mathrm{ac}=\mathrm{b}(\mathrm{a}+\mathrm{c})$

$\Rightarrow 2\mathrm{ac}=15\ \mathrm{b}\ldots(1)$

$\Rightarrow 2\mathrm{ac}=15\left(\frac{36}{5}\right)=108$

$\Rightarrow \mathrm{ac}=54$

$\mathrm{a}+\mathrm{c}=15$

$\mathrm{a}^2+\mathrm{c}^2+2\mathrm{ac}=225$

$\mathrm{a}^2+\mathrm{c}^2=225-108=117$

Q5. $\left(x^2-9x+11\right)^2-(x-4)(x-5)=3$

(1) $\left(x^2-9x+11\right)^2-\left(x^2-9x+20\right)=3$

Let $x^2-9x+11=t$

$t^2-(t+9)=3$

$\Rightarrow t^2-t-12=0$

$\Rightarrow t^2-4t+3t-12=0$

$\Rightarrow t(t-4)+3(t-4)=0$

$\Rightarrow t=4$ or -3

$x^2-9x+11=4$

$x^2-9x+7=0$

Here, we will get irrational roots

$x^2-9x+11=-3$

$x^2-9x+14=0$

$x^2-7x-2x+14=0$

$\Rightarrow x=7,2$

$\Rightarrow$ Product of all rational roots $=14$

Q6. $2x^2+(\cos\theta)x-1=0$

(2) $\alpha_\theta+\beta_\theta=\frac{-\cos\theta}{2}$

$\alpha_\theta\cdot\beta_\theta=\frac{-1}{2}$

$\alpha_\theta^2+\beta_\theta^2=(\alpha_\theta+\beta_\theta)^2-2\alpha_\theta\beta_\theta\frac{\cos^2\theta}{4}+1$

$\alpha_\theta^4+\beta_\theta^4=\left(a_\theta^2+\beta_\theta^2\right)^2-2\alpha_\theta^2\beta_\theta^2=\left(\frac{\cos^2\theta}{4}+1\right)^2-\frac{2}{4}$

$=\left(\frac{\cos^2\theta}{4}+1\right)^2-\frac{1}{2}$

Maximum when $\cos\theta=1$

$$M=\left(\frac{1}{4}+1\right)^2-\frac{1}{2}$$

$$M=\frac{17}{16}$$

Minimum when $\cos\theta=0$

$$m=1-\frac{1}{2}=\frac{1}{2}$$

$$16(M+m)=16\left(\frac{17}{16}+\frac{1}{2}\right)=25$$

FOCUS POINT mathongo

Q1. $|z| = 1$

(2) $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$

$\Rightarrow \left|z^2 + (\bar{z})^2\right| = 1$

Let $z = x + iy$

$\Rightarrow \left|(x+iy)^2 + (x-iy)^2\right| = 1$

$\Rightarrow \left|2x^2 - 2y^2\right| = 1$

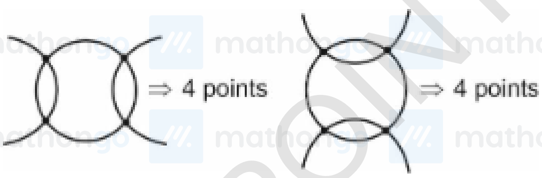
$\Rightarrow \left|x^2 - y^2\right| = \frac{1}{2}$

$\Rightarrow x^2 - y^2 = \frac{\pm 1}{2}$

and $x^2 + y^2 = 1$

Case I: $x^2 - y^2 = \frac{1}{2}$

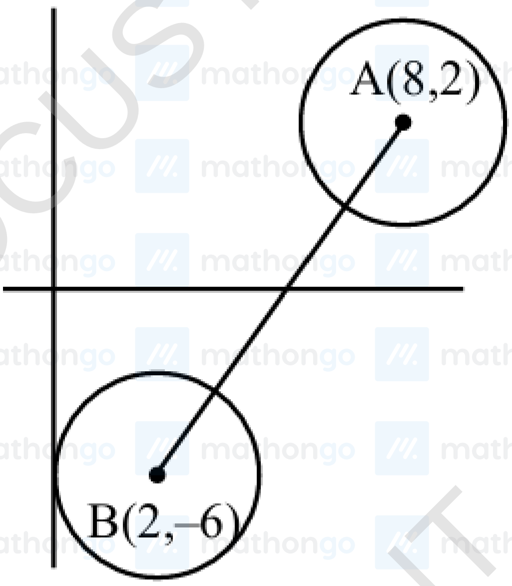
Case II: $x^2 - y^2 = -\frac{1}{2}$



Hence, we get 8 complex number

Q2.

(4)



$\because AB = \sqrt{100} = 10$

$\therefore |Z_1 - Z_2|_{\min} = 10 - 2 - 1 = 7$

Q3. $\alpha + \beta = a \quad \alpha\beta = -b$

(31) $P_6 = aP_5 + bP_4$

$45\sqrt{7}i = a \times 11\sqrt{7}i + b(-3\sqrt{7})i$

$45 = 11a - 3\,b$

and

$P_5 = \mathrm{aP}_4 + bP_3$
$11\sqrt{7}\mathrm{i} = \mathrm{a}(-3\sqrt{7}\mathrm{i}) + \mathrm{b}(-5\sqrt{7}\mathrm{i})$
$11 = -3\mathrm{a} - 5\ \mathrm{b}$
$\mathrm{a} = 3,\ \mathrm{b} = -4$
$|\alpha^4 + \beta^4| = \sqrt{(\alpha^4 - \beta^4)^2 + 4\alpha^4\beta^4}$
$= \sqrt{-63 + 4.4^4}$
$= \sqrt{-63 + 1024} = \sqrt{961} = 31$

Q4. $x^2 - (3 - 2\mathrm{i})\mathrm{x} - (2\mathrm{i} - 2) = 0$

(4) $\mathrm{x} = \dfrac{(3 - 2\mathrm{i}) \pm \sqrt{(3 - 2\mathrm{i})^2 - 4(1)(-(2\mathrm{i} - 2))}}{2(1)}$

$== \dfrac{(3 - 2\mathrm{i}) \pm \sqrt{9 - 4 - 12\mathrm{i} + 8\mathrm{i} - 8}}{2}$

$== \dfrac{3 - 2\mathrm{i} \pm \sqrt{-3 - 4\mathrm{i}}}{2}$

$= \dfrac{3 - 2\mathrm{i} \pm \sqrt{(1)^2 + (2\mathrm{i})^2 - 2(1)(2\mathrm{i})}}{2}$

$= \dfrac{3 - 2\mathrm{i} \pm (1 - 2\mathrm{i})}{2}$

$\Rightarrow \dfrac{3 - 2\mathrm{i} + 1 - 2\mathrm{i}}{2}$ or $\dfrac{3 - 2\mathrm{i} - 1 + 2\mathrm{i}}{2}$

$\Rightarrow 2 - 2\mathrm{i}$ or $1 + 0\mathrm{i}$

So $\alpha\gamma + \beta\delta = 2(1) + (-2)(0) = 2$

Q5. $|z| = 1$

(2) $\arg(z_1) = -\dfrac{\pi}{4}, \arg(z_2) = 0, \arg(z_3) = \dfrac{\pi}{4}$

$z_1 = |1|e^{-\frac{\pi}{4}} = \dfrac{1}{\sqrt{2}} - \dfrac{i}{\sqrt{2}}$

$z_2 = 1 + 0i$

$z_3 = \dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}$

$z_1\bar{z}_2 = \dfrac{1 - i}{\sqrt{2}}$

$z_2\bar{z}_3 = \dfrac{1 - i}{\sqrt{2}}$

$z_3\bar{z}_1 = \dfrac{(1 + i)^2}{2}$

$z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1 = \sqrt{2} + i(1 - \sqrt{2})$

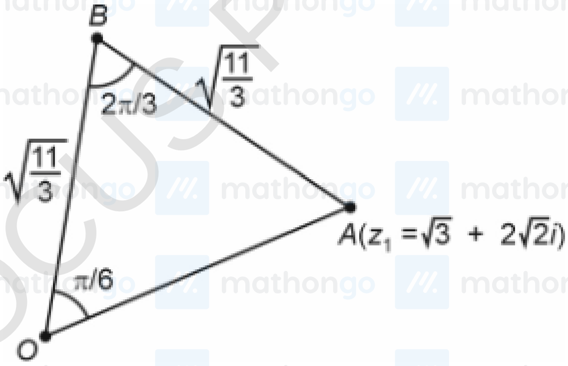
$|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = 5 - 2\sqrt{2}$

$\alpha = 5, \beta = -2$

$\alpha_2^2 + \beta^2 = 29$

Q6.

(2)



$OA = |z_1| = \sqrt{3+8} = \sqrt{11}$

and $OB = \frac{1}{\sqrt{3}}|z_1| = \sqrt{\frac{11}{3}}$

$AB^2 = OA^2 + OB^2 - 2 \cdot OA \cdot OB \cos\frac{\pi}{6}$

$= 11 + \frac{11}{3} - 2 \cdot \frac{11}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}$

$\therefore \quad AB = \sqrt{\frac{11}{3}}$

$\therefore \quad$ Area of $\triangle ABD = \frac{1}{2} \cdot OA \cdot OB \cdot \sin\frac{\pi}{6}$

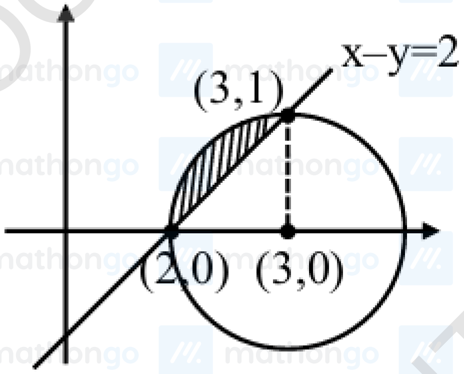
$= \frac{11}{4\sqrt{3}}$ sq. units

Here $OB = AB$ and $\angle A = \frac{2\pi}{3}$

$\therefore \quad \triangle ABD$ is an obtuse angled isosceles triangle.

Q7.

(1)



Let $z = x + iy$

$(x+iy)(1+i) + (x-iy)(1-i) = 4$

$x + ix + iy - y + x - ix - iy - y = 4$

$2x - 2y = 4$

$x - y = 2$

$|z-3| \leq 1$

$(x-3)^2 + y^2 \leq 1$

Area of shaded region $= \frac{\pi \cdot 1^2}{4} - \frac{1}{2} \cdot 1 \cdot 1 = \frac{\pi}{4} - \frac{1}{2}$

Area of unshaded region inside the circle

$= \frac{3}{4}\pi \cdot 1^2 + \frac{1}{2} \cdot 1 \cdot 1 = \frac{3\pi}{4} + \frac{1}{2}$

$\therefore$ difference of area $= \left(\frac{3\pi}{4} + \frac{1}{2}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{2} + 1$

Q8. Let $z = x + iy \Rightarrow \bar{z} = x - iy$

(2) $3|\bar{z} - i| = 1|2\bar{z} + i|$

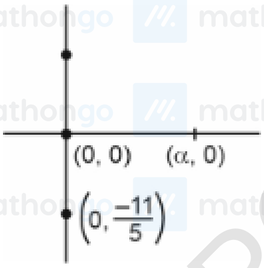
$= 3 \mid (x - (y+1)i| = |2x + i(1-2y)|$

$= 3\sqrt{x^2 + (y+1)^2} = \sqrt{(2x)^2 + (1-2y)^2}$

$= 9\left(x^2 + y^2 + 2y + 1\right) = 4x^2 + 4y^2 - 4y + 1$

$\Rightarrow 5x^2 + 5y^2 + 22y + 8 = 0$

$\Rightarrow$ Centre $\equiv \left(0, -\frac{11}{5}\right)$



Area of Δ

$= \frac{1}{2}|\alpha|\left|\frac{-11}{5}\right| = 11$

$\Rightarrow \quad |\alpha| = 10$

$\Rightarrow \quad \alpha^2 = 100$

Q9. $2z^2 - 3z - 2i = 0$ ...(i)

(1) $2\left(z - \frac{i}{z}\right) = 3$

As α, β are roots of (i)

$\alpha - \frac{i}{\alpha} = \frac{3}{2}$

$\Rightarrow \alpha^2 - \frac{1}{\alpha^2} - 2i = \frac{9}{4}$

$\Rightarrow \alpha^2 - \frac{1}{\alpha^2} = \frac{9}{4} + 2i$

Squaring both sides

$\Rightarrow \alpha^4 + \frac{1}{\alpha^4} - 2 = \frac{81}{16} - 4 + 9i$

$\Rightarrow \alpha^4 + \frac{1}{\alpha^4} = \frac{49}{16} + 9i$

Similarly, $\beta^4+\frac{1}{\beta^4}=\frac{49}{16}+9i$

$$\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}$$

$$=\frac{\alpha^{15}\left(\alpha^4+\frac{1}{\alpha^4}\right)+\beta^{15}\left(\beta^4+\frac{1}{\beta^4}\right)}{\alpha^{15}+\beta^{15}}$$

$$=\frac{49}{16}+9i$$

$$\operatorname{Re}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)=\frac{49}{16}$$

$$\operatorname{Im}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)=9$$

$$\Rightarrow 16\operatorname{Re}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)\cdot \operatorname{lm}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)$$

$$=16\times\frac{49}{16}\times 9$$

$$=441$$

Q10. $a, b\in I, -3\le a, b\le 3, a+b\ne 0$

(10) $|z-a|=|z+b|$

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1$$

$$\Rightarrow \begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1$$

$$\Rightarrow z\begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1$$

$$\Rightarrow z\begin{vmatrix} 1 & 0 & 0 \\ \omega & z+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & z+\omega-\omega^2 \end{vmatrix}=1$$

$$\Rightarrow z^3=1$$

$$\Rightarrow z=\omega, \omega^2, 1$$

Now

$|1-\mathrm{a}|=|1+\mathrm{b}|$

$\Rightarrow 10$ pairs

Q1. (1) $a = 3$

$S_4 = \frac{1}{5}(S_8 - S_4)$

$\Rightarrow 5 S_4 = S_8 - S_4$

$\Rightarrow 6 S_4 = S_8$

$\Rightarrow 6 \cdot \frac{4}{2}[2 \times 3 + (4-1) \times d]$

$= \frac{8}{2}[2 \times 3 + (8-1)d]$

$\Rightarrow 12(6 + 3d) = 4(6 + 7d)$

$\Rightarrow 18 + 9d = 6 + 7d$

$\Rightarrow d = -6$

$S_{20} = \frac{20}{2}[2 \times 3 + (20-1)(-6)]$

$= 10[6 - 114]$

$= -1080$

Q2. (474) $S_{11} = \frac{11}{2}(2a + 10d) = 88$

$a + 5d = 8$

$a = 8 - 5 \times \frac{3}{2} = \frac{1}{2}$

Roots are

$T_{10} = a + 9d = \frac{1}{2} + 9 \times \frac{3}{2} = 14$

$T_{11} = a + 10d = \frac{1}{2} + 10 \times \frac{3}{2} = \frac{31}{2}$

$\frac{p}{3} = T_{10} + T_{11} = 14 + \frac{31}{2} = \frac{59}{2}$

$p = \frac{177}{2}$

$\frac{q}{3} = T_{10} \times T_{11} = 7 \times 31 = 217$

$q = 651$

$q - 2p$

$= 651 - 177$

$= 474$

Q3. (3) $S_{40} = 1030 \Rightarrow \frac{40}{2}[2a + 39d] = 1030$

$\Rightarrow \quad 2a + 39d = \frac{103}{2} \ldots .(1)$

$S_{12} = 57 \Rightarrow \frac{12}{2}[2a + 11d] = 57$

$\Rightarrow \quad 2a + 11d = \frac{57}{6} \ldots (2)$

Equation (1) - equation (2)

$$28d=\frac{103}{2}-\frac{57}{6}$$
$$28d=\frac{309-57}{6}$$
$d=\frac{3}{2}$
$\Rightarrow \quad a=-\frac{7}{2}$
$S_{30}-S_{10}=\frac{30}{2}[2a+29d]-\frac{10}{2}[2a+9d]$
$=15[2a+29d]-5[2a+9d]$
$=5[6a+87d-2a-9d]$
$=5[4a+78d]$
$=5[-14+117]$
$=515$

Q4. (1) $\mathrm{S}_3=3\mathrm{a}+3\mathrm{~d}=54$
$\Rightarrow \mathrm{a}+\mathrm{d}=18$
$\mathrm{S}_{20}=10(2\mathrm{a}+19\mathrm{~d})$
$\Rightarrow 10(36+17\mathrm{~d})$
$\Rightarrow 1600<10(36+17\mathrm{~d})<1800$
$\Rightarrow 160<36+17\mathrm{~d}<180$
$\Rightarrow 124<17\mathrm{~d}<144$
$$\Rightarrow 7\frac{5}{17}<\mathrm{d}<8\frac{8}{17}$$
Common difference will be natural number
$\Rightarrow d=8 \Rightarrow a=10$
$\Rightarrow a_{11}=10+10\times 8=90$

Q5. (125) No, of 3 digits $=999-99=900$
No. of 3 digit numbers divisible by 2 \& 3 i.e. by 6
$\frac{900}{6}=150$
No. of 3 digit numbers divisible by 4 \& 9 i.e. by 36
$\frac{900}{36}=25$
$\therefore$ No of 3 digit numbers divisible by 2&3 but not by 4&9
$150-25=125$

Q6. (2)
$$\mathrm{T_m}=\frac{1}{25},\ \mathrm{T}_{25}=\frac{1}{20}, 20\sum_{\mathrm{r}=1}^{25}\mathrm{T_r}=13$$
$$\mathrm{T_m}=\mathrm{a}+(\mathrm{m}-1)\mathrm{d}=\frac{1}{25}\ldots\ldots.(1)$$
$$\mathrm{T}_{25}=\mathrm{a}+24\mathrm{~d}=\frac{1}{20}$$
$$20\cdot\frac{25}{2}\left[\mathrm{a}+\frac{1}{20}\right]=13\Rightarrow \mathrm{a}=\frac{1}{500}$$
also, $20\ \mathrm{S}_{25}=20\cdot\frac{25}{2}[2\mathrm{a}+24\mathrm{~d}]=13 \Rightarrow\ \mathrm{d}=\frac{1}{500}$
from (1) $\frac{1}{500}+\frac{\mathrm{m}-1}{500}=\frac{1}{25} \Rightarrow\ \mathrm{m}=20$

Now,

$5 \mathrm{~m} \sum_{\mathrm{r}=\mathrm{m}}^{2 \mathrm{~m}} \mathrm{~T}_{\mathrm{r}}=100 \sum_{\mathrm{r}=20}^{40} \mathrm{~T}_{\mathrm{r}}=126$

Q7. $\frac{n}{2}[2a+(n-1)6]=(n-2)180^{\circ}$

(20) and $an+3n^2-3n=3n(n-2)180^{\circ}$...(i)

$\therefore$ Given $a+(n-1)6^{\circ}=219^{\circ}$

$\Rightarrow a=225^{\circ}-6n^{\circ}$

Putting value of a in (i)

We get $\left(225-6n^2\right)+3n^2-3n=180n-360^{\circ}$

$\Rightarrow 2n^2-42n-360=0$

$\Rightarrow n^2-14n-120=0$

$\Rightarrow (n-20)(n+6)=0$

$\Rightarrow n=20,-6$ (Rejected)

$\therefore n=20$

Q8. As $a_1+a_5+a_{10}+\ldots+a_{2020}+a_{2024}=2233$ ...(1)

(11132) We know in arithmetic progression.

Sum of terms equidistant from ends is equal

$\therefore$ from (1)

$$\underbrace{a_1+a_{2024}=a_5+a_{2020}=a_{10}+a_{2015}=\ldots}_{203 \text{ pairs}}$$

$\Rightarrow \quad 203\left(a_1+a_{2024}\right)=2233$

$\Rightarrow \quad a_1+a_{2024}=11$

Now $\sum_{i=1}^{2024} a_i=S_{2024}=\frac{2024}{2}\left[a_1+a_{2024}\right]$

$=1012(11)$

$=11132$

Q9. (4) $\mathrm{a}_1 \cdot \mathrm{a}_5=28 \Rightarrow \mathrm{a} \cdot \mathrm{ar}^4=28 \Rightarrow \mathrm{a}^2 \mathrm{r}^4=28 \quad \ldots(1)$

$a_2+a_4=29 \Rightarrow ar+ar^3=29$

$\Rightarrow \mathrm{ar}\left(1+\mathrm{r}^2\right)=29$

$\Rightarrow \mathrm{a}^2 \mathrm{r}^2\left(1+\mathrm{r}^2\right)^2=(29)^2 \quad \ldots(2)$

By Eq. (1) & (2)

$$\frac{r^2}{\left(1+r^2\right)^2}=\frac{28}{29 \times 29}$$

$$\Rightarrow \frac{r}{1+\mathrm{r}^2}=\frac{\sqrt{28}}{29} \Rightarrow r=\sqrt{28}$$

$\because a^2 r^4=28 \Rightarrow a^2 \times(28)^2=28$

$$\Rightarrow \mathrm{a}=\frac{1}{\sqrt{28}}$$

$$\therefore \mathrm{a}_6=\mathrm{ar}^5=\frac{1}{\sqrt{28}} \times(28)^2 \sqrt{28}=784$$

Q10. $S=a+(a+d)r+(a+2d)r^2+\ldots$

(2) Then $S=\frac{a}{1-r}+\frac{dr}{(1-r)^2},|r|<1$

Since, $r = \frac{1}{7}$ and $a = 5, d = \alpha$

$$7 = \frac{5}{1-\frac{1}{7}} + \frac{\alpha \cdot \frac{1}{7}}{\left(1-\frac{1}{7}\right)^2}$$

$\Rightarrow \alpha = 6$

Q11. $\mathrm{T_n} = \mathrm{S_n} - \mathrm{S_{n-1}}$

(2) $\Rightarrow \mathrm{T_n} = \frac{1}{8}(2\mathrm{n}-1)(2\mathrm{n}+1)(2\mathrm{n}+3)$

$$\Rightarrow \frac{1}{\mathrm{T_n}} = \frac{8}{(2\mathrm{n}-1)(2\mathrm{n}+1)(2\mathrm{n}+3)}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\mathrm{T_r}} = \lim_{n\to\infty} 8 \sum_{r=1}^{n} \frac{1}{(2\mathrm{n}-1)(2\mathrm{n}+1)(2\mathrm{n}+3)}$$

$$= \lim_{n\to\infty} \frac{8}{4} \sum \left(\frac{1}{(2\mathrm{n}-1)(2\mathrm{n}+1)} - \frac{1}{(2\mathrm{n}+1)(2\mathrm{n}+3)}\right)$$

$$= \lim_{n\to\infty} 2\left[\left(\frac{1}{1.3} - \frac{1}{3.5}\right) + \left(\frac{1}{3.5} - \frac{1}{5.7}\right) + \ldots\right]$$

$$= \frac{2}{3}$$

Q12.
(2)

$$S_n = \sum_{k=1}^{n} \frac{4}{K^2 + 5k + 6}$$

$$= \sum_{k=1}^{n} \frac{4}{(K+2)(K+3)} = 4\sum_{K=1}^{n}\left(\frac{1}{K+2} - \frac{1}{K+3}\right)$$

$$= 4\left[\frac{1}{3} - \frac{1}{4}\right]$$

$$= 4\left[\frac{1}{4} - \frac{1}{5}\right]$$

$$= 4\left[\frac{1}{n+2} - \frac{1}{n+3}\right]$$

$$S_n = 4\left[\frac{1}{3} - \frac{1}{n+3}\right]$$

$$S_{2025} = 4\left[\frac{1}{3} - \frac{1}{2028}\right]$$

$$S_{2025} = 4\left[\frac{675}{2028}\right]$$

$$507 S_{2025} = 675$$

Q13.
(4)

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^3+6k^2+11k+5}{(k+3)!}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^3+6k^2+11k+6-1}{(k+3)!}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{(k+1)(k+2)(k+3)-1}{(k+3)!}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{(k+1)(k+2)(k+3)}{(k+3)!}-\frac{1}{(k+3)!}$$

$$=\lim_{k=1}\sum_{k=1}^{n}\left(\frac{1}{k!}-\frac{1}{(k+3)!}\right)$$

$$=\lim_{k=1}\left(\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}\ldots+\frac{1}{n!}-\frac{1}{4!}-\frac{1}{5!}-\frac{1}{6!}\ldots.-\frac{1}{(n+3)!}\right)$$

$$=\frac{1}{1}+\frac{1}{2}+\frac{1}{6}=\frac{10}{6}=\frac{5}{3}$$

Q14.
(2)

Let the A.P. be

$a, a+2, a+2d, \ldots, a+(2k-1)d$

Now, $a+a+2d+a+4d+\ldots+a+(2k-2)d=40$

$ka+2d+4d+\ldots+(2k-2)d=40$

$\Rightarrow ka+\dfrac{k-1}{?}[2d+2kd-2d]=40$

$\Rightarrow ka+k(k-1)d=40$ ...(1)

And $a+d+a+3d+\ldots+a+(2k-1)d=55$

$\Rightarrow ka+\frac{k}{2}(d+2kd-d)=55$

$\Rightarrow ka+k^2d=55$ ....(2)

Also, $a+(2k-1)d-a=27$

$\Rightarrow \quad (2k-1)d=27 \Rightarrow d=\frac{27}{2k-1}$ ...(3)

From equation (1) and (2)

$k^2d-kd-k^2d=-15$

$\Rightarrow \quad d=\frac{15}{k}$

From equation (3) and (4)

$$\frac{27}{2k-1}=\frac{15}{k}$$

$27k=30k-15$

$\Rightarrow 3k=15$

$\Rightarrow k=5$

Q15.
(4)

$Sn = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} \ldots . N\text{ terms}$

$S_{2025} = \sum_{n=1}^{2025} \frac{1}{n(n+1)} = \sum_{n=1}^{2025} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) \quad \ldots \ldots \left(\frac{1}{2025} - \frac{1}{2026}\right)$

$= \frac{2025}{2026}$

$\sqrt{2026.\, S_{2025}} = \sqrt{2025} = 45$

Given : $\frac{6}{2}[-2p + (6-1)p] = 45$

$9p = 45$

$p = 5$

$|A_{20} - A_{15}| = |-5 + 19 \times 5| - [-5 + 14 \times 5]$

$= |90 - 65|$

$= 25$

Q16.
(2)

$a_0 = 0, a_1 = \frac{1}{2}$

$2a_{n+2} = 5a_{n+1} - 3a_n$

$2x^2 - 5x + 3 = 0 \Rightarrow x = 1, 3/2$

$\therefore a_n = Al^n + B\left(\frac{3}{2}\right)^n$

$\left.\begin{array}{ll} n = 0 & 0 = A + B \\ n = 1 & \frac{1}{2} = A + \frac{3}{2}B \end{array}\right] \begin{array}{l} A = -1 \\ B = 1 \end{array}$

$\Rightarrow a_n = -1 + \left(\frac{3}{2}\right)^n$

$\sum_{k=1}^{100} a_k = \sum_{k=1}^{100} (-1) + \left(\frac{3}{2}\right)^k$

$= -100 + \frac{\left(\frac{3}{2}\right)\left(\left(\frac{3}{2}\right)^{100} - 1\right)}{\frac{3}{2} - 1}$

$= -100 + 3\left(\left(\frac{3}{2}\right)^{100} - 1\right)$

$= 3 \cdot (a_{100}) - 100$

Q1. (3) A, K, N, P, R, U

A .... $5! = 120$

K .... $5! = 120$

N .... $5! = 120$

P A .... $4! = 24$

P K .... $4! = 24$

P N .... $4! = 24$

P R A ....$3! = 6$

P R K A N U = 1

P R K A U N = 1

Total $= 440$

$\Rightarrow 440^{\text{th}}$ word is P R K A U N

Q2. (3)

Group A		Group B		
B	G	B	G	Ways
4	1	0	3	${}^7C_4 \cdot {}^3C_1 \cdot {}^6C_0 \cdot {}^5C_3$
3	2	1	2	${}^7C_3 \cdot {}^3C_2 \cdot {}^6C_1 \cdot {}^5C_2$
2	3	2	1	${}^7C_2 \cdot {}^3C_3 \cdot {}^6C_2 \cdot {}^5C_1$

Total ways $= 30 \cdot {}^7C_4 + 180 \cdot {}^7C_3 + 75 \cdot {}^7C_2 = 8925$

Q3. (4)

Case I	$5 \ldots \ldots$	0
Case II	$5 \ldots \ldots$	
	5	1
	5	3
	6	0
	6	1
	6	2
	7	0
	7	1

$9 \times (8 \times 8 \times 8) = 4608$ but 50000 is not included, so total numbers $4608 - 1 = 4607$

Q4. (64) Let the number be

$2ab, a + b = 13$

$\Rightarrow a, b \in \{0, 9\}$

$\Rightarrow 6$ numbers $\{(9,4),(8,5)\ldots(4,9)\}$

Similarly, for $3ab, a + b = 12 \Rightarrow 7$ numbers

For $4ab, a + b = 11 \Rightarrow$ Numbers

For $5ab, a + b = 10 \Rightarrow 9$ numbers

For $6ab, a + b = 9 \Rightarrow 10$ numbers

For $7ab, a + b = 8 \Rightarrow 9$ numbers

For $8ab, a + b = 7 \Rightarrow 8$ numbers

For $9ab, a + b = 6 \Rightarrow 7$ numbers

$\therefore$ Total ways $= 64$.

Q5. (4) (i) number of numbers created using

$1111133 = \frac{7!}{5!2!} \Rightarrow 21$

(ii) number of numbers created using

$1111223 = \frac{7!}{4!2!} \Rightarrow 105$

(iii) number of numbers created using

$1112222 = \frac{7!}{4!3!} \Rightarrow 35$

Total $= 161$

Q6. (1405) (i) Single letter is used, then no. of words $= 5$

(ii) Two distinct letters are used, then no. of words

${}^5C_2 \times \left(\frac{6!}{2!4!} \times 2 + \frac{6!}{3!3!}\right) = 10(30 + 20) = 500$

(iii) Three distinct letters are used, then no. of words

${}^5C_3 \times \frac{6!}{2!2!2!} = 900$

Total no. of words $= 1405$

Q7. (1) $\underbrace{AB}_{12} \quad \underbrace{\text{MN........Z}}_{13}$

$= \underbrace{{}^{12}C_2}_{\text{Selection of two leters before M}} \times \underbrace{{}^{13}C_2}_{\text{Selection of two leter after M}} = 5148$

Q8. (2) Total - when B_1 and B_2 are together

$= 2!(3!4!) - 2!(3!(3!2!)) = 144$

Q9. (1) DAUGHTER

Total words $= 8\,!$

Total words in which vowels are together $= 6! \times 3!$ words in which all vowels are not together

$= 8! - 6! \times 3!$

$= 6![56 - 6]$

$= 720 \times 50$

$= 36000$

Q10. (17280) A : number of ways that all boys sit together $= 5! \times 5!$

B : number of ways if no 2 boys sit together $= 4! \times 5\,!$

$A \cap B = \phi$

Required no. of ways $= 5! \times 5! + 4! \times 5! = 17280$

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Q1. The general term of multinomial expansion is

(612) $\frac{6!}{\alpha!\beta!\gamma!}(1)^{\alpha}\left(2^{\frac{1}{3}}\right)^{\beta}\left(3^{\frac{1}{2}}\right)^{\gamma}$

For terms to be rational $3 \mid \beta$ and $2 \mid \gamma$

β	γ	α	Term
0	0	6	1
0	2	4	$15.3 = 45$
0	4	2	$15.3^2 = 135$
0	6	0	$1.3^3 = 27$
3	0	3	$20.2 = 40$
3	2	1	$60.2.3 = 360$
6	0	0	$1.4 = 4$

$\Rightarrow$ Sum of rational terms

$= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612$

Q2. $(1+x)^{n+4}$

(3) ${}^{n+4}C_4, {}^{n+4}C_5, {}^{n+4}C_6, \rightarrow$ A.P.

$\Rightarrow 2 \times \times {}^{n+4}C_5 = {}^{n+4}C_4 + {}^{n+4}C_6$

$\Rightarrow 4 \times {}^{n+4}C_5 = \left({}^{n+4}C_4 + {}^{n+4}C_5\right) + \left({}^{n+4}C_5 + {}^{n+4}C_6\right)$

$\Rightarrow 4 \times {}^{n+4}C_5 = {}^{n+5}C_5 + {}^{n+5}C_6$

$\Rightarrow 4 \times \frac{(n+4)!}{5! \cdot (n-1)!} = \frac{(n+6)!}{6! \cdot n!}$

$\Rightarrow 4 = \frac{(n+6)(n+5)}{6n}$

$\Rightarrow n^2 + 11n + 30 = 24n$

$\Rightarrow n^2 - 13n + 30 = 0$

$\Rightarrow n = 3, 10(\text{ rejected })$

$\because n \neq 10$

$\therefore$ Largest binomial coefficient in expansion of

$(1+x)^7$

$(\because n + 4 = 7)$

is coeff. of middle term

$\Rightarrow {}^7C_4 = {}^7C_3 = 35$

Q3. $A = {}^{2n-1}C_{29} \quad B = {}^{2n-1}C_{11}$

(3) $2\,{}^{2n-1}C_{29} = 5\,{}^{2n-1}C_{11}$

$$2\frac{(2n-1)!}{29!(2n-30)!} = 5\frac{(2n-1)!}{(2n-12)!11!}$$

$$\frac{1}{29\ldots 12\cdot 5} = \frac{1}{(2n-12)(2n-13)\ldots(2n-29)2}$$

$$\frac{1}{30\cdot 29\ldots 12} = \frac{1}{(2n-12)(2n-13)\ldots(2n-29)12}$$

$2n-12 = 30$

$n = 21$

Q4. $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

(1) $-\left[{}^5C_0x^5 + {}^5C_1x^4\left(\sqrt{x^3-1}\right) + \ldots\ldots + {}^5C_5\left(\sqrt{x^3-1}\right)^5\right] +$

$\left[{}^5C_0x^5 - {}^5C_1x^4\left(\sqrt{x^3-1}\right) + \ldots\ldots\ldots + {}^{-5}C_5\left(\sqrt{x^3-1}\right)^5\right]$

$= 2\left[x^5 + {}^5C_2x^3\left(x^3-1\right) + {}^5C_4\left(x^3-1\right)^2\right]$

$= 10x^7 + 20x^6 + 2x^5 - 20x^4 - 20x^3 + 10x$

Now

$\alpha = 10, \beta = 2, \gamma = -20, \delta = 10$

Also,

$$\left.\begin{array}{r}10u+2v=18\\-20u+10v=20\end{array}\right\} u=1, v=4$$

$u+v=5$

Q5. $(1+x)^p(1-x)^q = \left({}^pC_0 + {}^pC_1x + {}^pC_2x^2 + \ldots\right)\left({}^qC_0 - {}^qC_1x + {}^qC_2x^2 + \ldots\right)$

(2) coff of $x \equiv {}^pC_0{}^qC_1 + {}^pC_1{}^qC_0 = 1$

$p - q = 1$

coff of $x^2 \equiv {}^pC_0{}^qC_2 - {}^pC_1{}^qC_1 + {}^pC_2{}^qC_0 = -2$

$$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -2$$

$q^2 - q - 2pq + p^2 - p = -4$

$1 - (p+q) = -4$

$p + q = 5$

$p = 3$

$q = 2$

so $p^2 + q^2 = 13$

Q6. Coefficient of

(1) $T_r, T_{r+1}, T_{r+2} \to GP$

$\Rightarrow \left({}^{12}C_r\right)^2 = {}^{12}C_{r-1}\cdot{}^{12}C_{r+1}$

$\Rightarrow \left({}^{12}C_r\right)^2 = {}^{12}C_{r-1}\cdot{}^{12}C_{r+1}$

but no three consecutive binomial coefficient are in

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$\Rightarrow P=0$

Now for $\left(3^{1/4}+4^{1/3}\right)^{12}, T_{r+1}={}^{12}C_r(4)^{K/3}(3)^{\frac{12-K}{4}}$

for rational terms $\mathrm{K}=0,12$

sum of rational terms

$={}^{12}\mathrm{C}04^0\cdot 3^3+{}^{12}\mathrm{C}_{12}\cdot 4^4\cdot 3^0$

$=27+256=283=q$

$\therefore p+q=283$

Q7. General term $={}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}\left\{7^{1/3}\right\}^{\mathrm{n-r}}\left(11^{1/12}\right)^{\mathrm{r}}$

(1) $={}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}\{7\}^{\frac{\mathrm{n-r}}{3}}(11)^{\mathrm{r}/12}$

For integral terms, r must be multiple of 12

$\therefore \mathrm{r}=12\mathrm{k}, \mathrm{k}\in \mathrm{W}$

Total values of $r=183$

Hence $\max r=12(182)$

$=2184$

Min value of $n=2184$

Q8. $7^{103}=7\left(7^{102}\right)=7(343)^{34}=7(345-2)^{34}$

(4) $7^{103}=23\,\mathrm{K}_1+7.2^{34}$

Now $7.2^{34}=7.2^2\cdot 2^{32}$

$=28\cdot(256)^4$

$=28(253+3)^4$

$\therefore 28\times 81\Rightarrow(23+5)(69+12)$

$23\,\mathrm{K}_2+60$

$\therefore$ Remainder $=14$

Q9. $(1+x)^{11}={}^{11}C_0+{}^{11}C_1x+{}^{11}C_2x^2+\cdots+{}^{11}C_{11}x^{11}$

(2035) $\int_0^1(1+x)^{11}dx=\int_0^1\left({}^{11}C_0+{}^{11}C_1x+{}^{11}C_2x^2+\cdots+{}^{11}C_{11}xx^{11}\right)dx$

$$\left.\frac{(1-x)^{12}}{12}\right|_0^1={}^{11}C_{0x}+\frac{{}^{11}C_1x^2}{2}+\frac{{}^{11}C_2x}{3}+\cdots+\left.\frac{{}^{11}C_{11}x^{12}}{12}\right]_0^1$$

$$\frac{2^{12}-1}{12}=C_0+\frac{C_1}{2}+\frac{C_2}{3}+\cdots+\frac{C_{11}}{12}\ldots(1)$$

Now,

$$\int_{-1}^{0}(1+x)^{11}dx=\int_{-1}^{0}\left({}^{11}C_0+{}^{11}C_1x+{}^{11}C_2x^2+\cdots+{}^{11}C_{11}x^{11}\right)dx$$

$$\left.\frac{(1+x)^{12}}{12}\right]_{-1}^{0}=\left.{}^{11}C_0x+\frac{{}^{11}C_1x^2}{2}+\frac{{}^{11}C_2x^3}{3}+\cdots+\frac{{}^{11}C_{11}x^{12}}{12}\right]_{-1}^{0}$$

$\frac{1}{12}=C_0-\frac{C_1}{2}+\frac{C_2}{3}\cdots$

$(1)-(2)$

$$=\frac{2^{12}-2}{12}=2\left[\frac{C_1}{2}+\frac{C_3}{4}+\cdots\right]$$

$$\Rightarrow\sum_{r=0}^{5}\frac{C_{2r+1}}{2r+2}=\frac{2^{11}-1}{12}=\frac{2047}{12}=\frac{m}{n}$$

$=2047-12=2035$

Q10.
(5)

$$\alpha=1+\sum_{r=1}^{6}(-1)^{r-1}{}^{12}C_{2r-1}3^{r-1}$$

$$\alpha=1+\sum_{r=1}^{6}{}^{12}C_{2r-1}\frac{(\sqrt{3}i)^{2t-1}}{\sqrt{3}i}\quad i=\text{ iota, let }\sqrt{3}i=x$$

$$\alpha=1+\frac{1}{\sqrt{3}i}\left({}^{12}C_1x+{}^{12}C_3x^3+\ldots.{}^{12}C_{11}x^{11}\right)$$

$$=1+\frac{1}{\sqrt{3}i}\left(\frac{(1+\sqrt{3}i)^{12}-(1-\sqrt{3}i)^{12}}{2}\right)$$

$=1+\frac{1}{\sqrt{3}i}\left(\frac{(-2w^2)^{12}-(2w)^{12}}{2}\right)=1$

so distance of $(12,\sqrt{3})$ from $x-\sqrt{3}y+1=0$ is

$\frac{12-3+1}{2}=5$

Q11.
(465)

$$\sum_{r=1}^{30}\frac{r^2\left({}^{30}C_r\right)^2}{{}^{30}C_{r-1}}$$

$$=\sum_{r=1}^{30}r^2\left(\frac{31-r}{r}\right)\cdot\frac{30!}{r!(30-r)!}$$

$$\left(\because\frac{{}^{30}C_r}{{}^{30}C_{r-1}}=\frac{30-r+1}{r}=\frac{31-r}{r}\right)$$

$$= \sum_{r=1}^{30} \frac{(31-r)30!}{(r-1)!(30-r)!}$$

$$= 30\sum_{r=1}^{30} \frac{(31-r)29!}{(r-1)!(30-r)!}$$

$$= 30\sum_{r=1}^{30} (30-r+1)^{29}C_{30-r}$$

$$= 30\left(\sum_{r=1}^{30} (31-r)^{29}C_{30-r} + \sum_{r=1}^{30} {}^{29}C_{30-r}\right)$$

$$= 30\left(29 \times 2^{28} + 2^{29}\right) = 30(29+2)2^{28}$$

$$= 15 \times 31 \times 2^{29}$$

$$= 465\left(2^{29}\right)$$

$$\alpha = 465$$

Q1.
(8788)
$8+(n-1)13=320$
$13n=325$
$n=25$
no. of terms $=25$

$$\text{mean} = \frac{\sum x_i}{n} = \frac{8+21+\ldots+320}{25} = \frac{\frac{25}{2}(8+320)}{25}$$

$$\text{variance } \sigma^2 = \frac{\sum x_i^2}{n} - (\text{ mean })^2$$

$$= \frac{8^2+21^2+\ldots.+320^2}{13} - (164)^2$$

$= 8788$

Q2.
(3)

$$\text{medain } = \ell + \left(\frac{\frac{N}{2}-F}{f}\right) \times h$$

$$= 12 + \left(\frac{\frac{N}{2}-18}{12}\right) \times 6 = 14$$

$$\Rightarrow \left(\frac{\frac{N}{2}-18}{12}\right) \times 6 = 2$$

$$\frac{N}{2} - 18 = 4 \Rightarrow N = 44$$

Q3.
(3)
Mean $\bar{x} = 5.5$

$$= \sum_{i=1}^{10} x_i = 5.5 \times 10 = 55$$

$$= \sum_{i=1}^{10} x_i^2 = 371$$

$$\left(\sum x_i\right)_{\text{new}} = 55 - (4+5) + (6+8) = 60$$

$$\left(\sum x_i\right)_{\text{new}} = 371 - \left(4^2+5^2\right) + \left(6^2+8^2\right) = 430$$

$$\text{Variance } \sigma^2 = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10}\right)^2$$

$$\sigma^2 = \frac{430}{10} - \left(\frac{60}{10}\right)^2$$

$\sigma^2 = 43 - 36$
$\sigma^2 = 7$

Q4.
(1)

$$\sum_{l=1}^{10}(x_l-2)=30$$

$$\sum_{i=1}^{10}x_l=50$$

$\Rightarrow$ Mean $=5$

$$\text{Variance }=\frac{4}{5}=\frac{\sum x_l^2}{10}-(\bar{x})^2$$

$$\frac{4}{5}=\frac{\sum x_l^2}{10}-25$$

$$\Rightarrow \sum x_l^2=258$$

Now, $\sum_{l=1}^{10}(x_l-\beta)^2=98$

$$\sum_{l=1}^{10}x_l^2-2\beta\sum_{l=1}^{10}x_l+10\beta^2=98$$

$\Rightarrow 258-2\beta(50)+10\beta^2=98$

$\Rightarrow 10\beta^2-100\beta+160=0$

$\Rightarrow \beta^2-10\beta+16=0$

$\Rightarrow \beta=8$ as $\beta>2$

Now, as per the question

$2(x_1-1)+4\beta, 2(x_2-1)+4\beta, \ldots 2(x_{10}-1)+4\beta$

Can be simplified as

$2x_1+30, 2x_2+30, \ldots, 2x_{10}+30$

$\mu=2(5)+30=40$

$$\sigma^2=2^2\left(\frac{4}{5}\right)=\frac{16}{5}$$

$$\frac{\beta\mu}{\sigma^2}=\frac{8\times 40}{\frac{16}{5}}=100$$

Q1.
(1)

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\therefore \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\therefore \quad b = 0, e = 0, h = 1$

and $\begin{bmatrix} a & 0 & c \\ d & 0 & f \\ g & 1 & i \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\left.\begin{array}{l} 4a + 3c = 0 \\ \therefore\ 4d + 3f = 1 \\ 4g + 1 + 3i = 0 \end{array}\right\} \ldots .(1)$$

and $\begin{bmatrix} a & 0 & c \\ d & 0 & f \\ g & 1 & i \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\left.\begin{array}{l} 2a + 2c = 1 \\ \therefore 2d + 2f = 0 \\ 2g + 1 + 2i = 0 \end{array}\right\} \ldots (2)$$

From equation (1) and (2) we get

$d = 1, f = -1$

$\therefore \quad a_{23} = -1$

Q2.
(2)

$$A = \begin{bmatrix} (\sqrt{2})^2 & (\sqrt{2})^3 & (\sqrt{2})^4 \\ (\sqrt{2})^3 & (\sqrt{2})^4 & (\sqrt{2})^5 \\ (\sqrt{2})^4 & (\sqrt{2})^5 & (\sqrt{2})^6 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}$$

$$A^2 = 2^2\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix}\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix}$$

$$= 4\begin{bmatrix} - & - & - \\ - & - & - \\ (2+4+8) & (2\sqrt{2}+4\sqrt{2}+8\sqrt{2}) & (4+8+16) \end{bmatrix}$$

Sum of elements of 3^{nd} row $= 4(14 + 14\sqrt{2} + 28)$

$= 4(42 + 14\sqrt{2})$

$= 168 + 56\sqrt{2}$

$\alpha + \beta\sqrt{2}$

$\therefore \alpha + \beta = 168 + 56 = 224$

Q3.
(4)

$\because \operatorname{tr}(A) = 3$ and $|A| = \frac{1}{2}$

Now, $B = \operatorname{adj}(\operatorname{adj}(2A)) = |2A|^{3-2} \cdot (2A)$

$= 2^3|A| \cdot 2A = 8A$

$\therefore \operatorname{tr}(B) = 8\operatorname{tr}(A) = 24$

and $|B| = |8A| = 8^3 \cdot \frac{1}{2} = 256$

$\therefore \quad \operatorname{trace}(B) + |B| = 24 + 256$

$= 280$

Q4. (1613) $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

No. of elements in $\mathrm{S}_1 : \mathrm{A} = \mathrm{A}^{\mathrm{T}} \Rightarrow 5^3 \times 5^3$

No. of elements in $A = -A^T \Rightarrow 0$

since no. zero in 5

No. of elements in $\mathrm{S}_3 \Rightarrow$

$$\left.\begin{array}{c} a_{11} + a_{22} + a_{33} = 0 \Rightarrow (1,2,-3) \Rightarrow 31 \\ \text{or} \\ (1,1,-2) \Rightarrow 3 \\ \text{or} \\ (-1,-1,2) \Rightarrow 3 \end{array}\right\} \Rightarrow 12 \times 5^6$$

$\mathrm{n}(\mathrm{S}_1 \cap \mathrm{S}_3) = 12 \times 5^3$

$\mathrm{n}(\mathrm{S}_1 \cup \mathrm{S}_2 \cup \mathrm{S}_3) = 5^6(1+12) - 12 \times 5^3$

$\Rightarrow 5^3 \times [13 \times 5^3 - 12] = 125\alpha$

$\alpha = 1613$

Q5. (2) $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

Now finding characteristic equation

$\begin{vmatrix} 2-\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)(-\lambda) - (-1)(1) = -2\lambda + \lambda^2 + 1 = 0$

$\Rightarrow \lambda^2 - 2\lambda + 1 = 0$

$\Rightarrow (\lambda - 1)^2 = 0$

$\Rightarrow \lambda = 1$

Since A satisfies $(A - I)^2 = 0$

$\therefore \quad A = I + N$ where

$N = A - I$

$N = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

$N^2 = 0$

$A^m = (I+N)^m = I + mN$

$A^m \cdot A^m = (I+mN)(I+mN) = I + 2mN + m^2N^2$

Since $N^2 = 0$

$\Rightarrow \quad A^{m^2} = I + 2mN$

Now putting in given condition

$I+m^2N+I+mN=3I-A^{-6}$

$A^{-1}=\begin{bmatrix}0 & 1\\ -1 & 2\end{bmatrix}$

$A^{-6}=\left(A^{-1}\right)^6=I+(-6)N$

$\therefore$ Putting in (i)

$\left(m^2+m\right)N=I-(I-6N)$

$\left(m^2+m\right)N=6N$

Since $N\neq 0$

$\Rightarrow m^2+m=6$

$\Rightarrow m^2+m-6=0$

$\Rightarrow (m-2)(m+3)=0$

$\Rightarrow m=2,-3$

$\therefore$ Number of elements in S is 2

Q6. As $A\,\mathrm{adj}\,A=|A|l, \det(\lambda A)=\lambda^n\det A$

(34) $\det(3\,\mathrm{adj}(-6\,\mathrm{adj}(3A)))=3^3\det(\mathrm{adj}(-6\,\mathrm{adj}(3A)))$

$=3^3(-6\,\mathrm{adj}(3A))^2$

$=3^3(-6)^6|3A|^4$

$=3^9 2^6\cdot 3^{12}\cdot(-2)^4$

$=3^{21}\cdot 2^{10}$

Now comparing with given condition

$2^{m+n}3^{mn}=2^{10}\cdot 3^{21}$

$m+n=10, mn=21$

$\Rightarrow \quad m=7, n=3(m>n)$

$\therefore \quad 4m+2n=28+6=34$

Q7.
(44)

$X^{\top}AX=0$

$(xyz)\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=0$

$(xyz)\begin{bmatrix}a_1x+a_2y+a_3z\\ b_1x+b_2y+b_3z\\ c_1x+c_2y+c_3z\end{bmatrix}=0$

$x\left(a_1x+a_2y+a_3z\right)+y\left(b_1x+b_2y+b_3z\right)$
$+z\left(c_1x+c_2y+c_3z\right)$

$a_1=0, b_2=0, c_3=0$
$a_2+b_1=0, a_3+c_1=0, b_3+c_2=0$
$A=$ skew symmetric matrix

$A=\begin{bmatrix}0 & x & y\\ -x & 0 & z\\ -y & -z & 0\end{bmatrix}; A\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}=\begin{bmatrix}1\\ 4\\ -5\end{bmatrix}$

$\Rightarrow\begin{bmatrix}0 & x & y\\ -x & 0 & z\\ -y & -z & 0\end{bmatrix}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}=\begin{bmatrix}1\\ 4\\ -5\end{bmatrix}$

$x+y=1$
$-x+z=4$
$y+z=5$

$\begin{bmatrix}0 & x & y\\ -x & 0 & z\\ -y & -z & 0\end{bmatrix}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}=\begin{bmatrix}1\\ 4\\ -8\end{bmatrix}$

$2x+y=0 \quad x=-1$
$-x+z=4 \quad y=2$
$-y-2z=-8 \quad z=3$

$A=\begin{bmatrix}0 & -1 & 2\\ 1 & 0 & 3\\ -2 & -3 & 0\end{bmatrix}$

$2(A+l)=\begin{bmatrix}2 & -2 & 4\\ 2 & 2 & 6\\ -2 & -6 & 2\end{bmatrix}$

$2(A+l)=120$
$\Rightarrow \det(\operatorname{adj}(2A+l))$
$=120^2=2^6\cdot 3^2\cdot 5^2$
$\therefore \quad \alpha=6, \beta=2, \gamma=2$
Hence $\alpha^2+\beta^2+\gamma^2=6^2+2^2+2^2=44$

Q8.
(3)

$|A| = \frac{11}{2}$

$C_{11} = \sum_{k=1}^{2} a_{1k} \cdot A_{1k} = a_{11} A_{11} + a_{12} A_{12} = |A| = \frac{11}{2}$

$C_{12} = \sum_{k=1}^{2} a_{1k} \cdot A_{2k} = 0$

$C_{21} = \sum_{k=1}^{2} a_{2k} \cdot A_{1k} = 0$

$C_{22} = \sum_{k=1}^{2} a_{2k} \cdot A_{2k} = |A| = \frac{11}{2}$

$C = \begin{bmatrix} 11/2 & 0 \\ 0 & 11/2 \end{bmatrix}$

$|C| = \frac{121}{4}$

$8|C| = 242$

Q9.
(4)

$\left[A\left(\operatorname{adj}\left(A^{-1}\right) + \operatorname{adj}\left(B^{-1}\right)\right)^{-1} \cdot B\right]^{-1}$

$B^{-1} \cdot \left(\operatorname{adj}\left(A^{-1}\right) + \operatorname{adj}\left(B^{-1}\right)\right) \cdot A^{-1}$

$B^{-1} \operatorname{adj}\left(A^{-1}\right) A^{-1} + B^{-1}\left(\operatorname{adj}\left(B^{-1}\right)\right) \cdot A^{-1}$

$B^{-1} \left|A^{-1}\right| I + \left|B^{-1}\right| I A^{-1}$

$\frac{B^{-1}}{|A|} + \frac{A^{-1}}{|B|}$

$\Rightarrow \frac{\operatorname{adj}B}{|B||A|} + \frac{\operatorname{adj}}{|A||B|}$

$= \frac{1}{|A||B|}(\operatorname{adj}B + \operatorname{adj}A)$

Q10.
(3)

$P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\because P^T P = I$

$B = PAPT$

Pre multiply by P^T (Given)

$P^T B = P^T P A P^T = A P^T$

Now post multiply by P

$P^T B P = A P^T P = A$

So $A^2 = \underbrace{P^T B P \;\; P^T}_{I} \; BP$

$A^2 = P^T B^2 P$

Similarly $A^{10} = P^T B^{10} P = C$

$A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ (Given)

$\Rightarrow A^2=\begin{bmatrix}\frac{1}{2} & -\sqrt{2}-2\\ 0 & 1\end{bmatrix}$

Similarly check A^3 and so on since $C=A^{10}\Rightarrow$ Sum of diagonal elements of C is $\left(\frac{1}{\sqrt{2}}\right)^{10}+1$

$=\frac{1}{32}+1=\frac{33}{32}=\frac{\mathrm{m}}{\mathrm{n}}$

$\mathrm{g\ cd(m,n)}=1$(Given)

$\Rightarrow \mathrm{m}+\mathrm{n}=65$

Q1.
(1) $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ \sin^2 x & 1+\cos^2 x & 4\sin 4x \\ \sin^2 x & \cos^2 x & 1+4\sin 4x \end{vmatrix}, x \in R$

$R_2 \to R_2 - R_1 \& R_3 \to R_3 \to R_1$

$f(x)\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$

Expand about R_1, use get

$f(x) = 2 + 4\sin 4x$

$\therefore$ M = max value of f(x) = 6

M = min value of $f(x) = -2$

$\therefore M^4 - M^4 = 1280$

Q2.
(3) $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & \lambda \end{vmatrix} = 0$

$\lambda = 17$

$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 9 \\ 1 & 5 & \mu \end{vmatrix} \neq 0$

$\mu \neq 18$

Q3.
(1) $\lim_{x\to 0}\begin{vmatrix} a+\frac{\sin x}{x} & 1 & b \\ a & 1+\frac{\sin x}{x} & b \\ a & 1 & b+\frac{\sin x}{x} \end{vmatrix} = \lambda + \mu a + vb$

At $\lim x \to 0$,

$f(x) = \begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1 \end{vmatrix} = \lambda + \mu a + vb$

$R_1 \to R_1 - R_2$
$R_2 \to R_2 - R_3$

$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ a & 1 & b+1 \end{vmatrix} = \lambda + \mu a + vb$

$C_2 \to C_1 - C_2$

$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ a & a+1 & b+1 \end{vmatrix} = \lambda + \mu a + vb$

$a + b + 2 = \lambda + \mu a + vb$

$\lambda = 2, \mu = 1, \quad v = 1$

$(\lambda + \mu + v) = (2+1+1)^2 = 16$

Q4. The given equations are
(1) $x+y+2z=6$
$2x+3y+az=a+1$
$-x-3y+bz=2b,$ where $a, b, \in \mathbf{R}$
For infinite many solutions:
$D=D_1=D_2=D_3=0$

$$\therefore D=\begin{vmatrix}1 & 1 & 2\\ 2 & 3 & a\\ -1 & -3 & b\end{vmatrix}=2a+b-6$$

$$D_1=\begin{vmatrix}6 & 1 & 2\\ a+1 & 3 & a\\ 2b & -3 & b\end{vmatrix}=12a+5b+ab-6$$

$$D_2=\begin{vmatrix}1 & 6 & 2\\ 2 & a+1 & a\\ -1 & 2b & b\end{vmatrix}=-4a-3b-ab+2$$

and $D_3=\begin{vmatrix}1 & 1 & 6\\ 2 & 3 & a+1\\ -1 & -3 & 2b\end{vmatrix}=2a+2b-16$

from above relations
$a=-2, b=10$
$\therefore 7a+3b=16$

Q5. $(\lambda-1)x+(\lambda-4)y+\lambda z=5$
(4) $\lambda x+(\lambda-1)y+(\lambda-4)z=7$
$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$
For infinitely many solutions

$$\mathrm{D}=\begin{vmatrix}\lambda-1 & \lambda-4 & \lambda\\ \lambda & \lambda-1 & \lambda-4\\ \lambda+1 & \lambda+2 & -(\lambda+2)\end{vmatrix}=0$$

$(\lambda-3)(2\lambda+1)=0$

$$\mathrm{D_x}=\begin{vmatrix}5 & \lambda-4 & \lambda\\ 7 & \lambda-1 & \lambda-4\\ 9 & \lambda+2 & -(\lambda+2)\end{vmatrix}=0$$

$2(3-\lambda)(23-2\lambda)=0$
$\lambda=3$
$\therefore \lambda^2+\lambda=9+3=12$

Q6. (1)

$2x - y + z = 4$
$5x + \lambda y + 3z = 12$
$100x - 47y + \mu z = 212$
$\Delta x = \Delta y = \Delta z = 0$

$$\Delta z = \begin{vmatrix} 2 & -1 & 4 \\ 5 & \lambda & 12 \\ 100 & -47 & 212 \end{vmatrix} = 0$$

$\Rightarrow 2(212\lambda + 564) + 1(1060 - 1200)$
$\Rightarrow 424\lambda + 1128 - 140 - 940 - 400\lambda = 0$
$\Rightarrow \lambda = -2$

$$\Delta y = \begin{vmatrix} 2 & -1 & 1 \\ 5 & -2 & 3 \\ 100 & -47 & \mu \end{vmatrix} = 0$$

$\Rightarrow 2(-2\mu + 141) + 1(5\mu - 300) + 1(-235 + 200) = 0$
$\Rightarrow \quad \mu = 53$
$\mu - 2\lambda = 53 - 2(-2) = 57$

Q7. (3)

$$\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 14 & 3 & \mu \end{vmatrix} = 0 \Rightarrow \lambda\mu + 42\lambda - 4\mu + 107 = 0$$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & -3 \\ 5 & \lambda & 5 \\ 33 & 3 & \mu \end{vmatrix} = 0 \Rightarrow 2\lambda\mu + 99\lambda - 10\mu + 255 = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 5 & 5 \\ 14 & 33 & \mu \end{vmatrix} = 0 \Rightarrow \mu = 13$$

Also, $\lambda = -1$
Hence, $\lambda + \mu = 13 - 1 = 12$

Q8. (4)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 1(20 - 16) - 1(10 - 4) + 1(4 - 2)$$

$= 4 - 6 + 2 = 0$

For infinite solutions
$\Delta_x = \Delta_y = \Delta_z = 0$
$m^2 - 3x + 2 = 0$
$m = 1, 2$
$\alpha = 1, \beta = 2$

$$\therefore \sum_{n=1}^{10} \left(n^\alpha + n^\beta\right) = \sum_{n=1}^{10} n^1 + \sum_{n=1}^{10} n^2$$

$$= \frac{10(11)}{2} + \frac{10(11)(21)}{6}$$

$= 55 + 385$
$= 440$

Q1. $A = [a_y]_{2\times 2}$ and entries are 0 or 1.

(3) $\therefore \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$

$\Rightarrow ad - bc = 0$

Case I: $ad = bc = 1$

$\therefore \quad a = b = c = d = 1$

Case II: $ad = bc = 0$

$a = 0, d = 0 \quad b = 0, c = 0$

$a = 0, d = 1 \quad b = 0, c = 1$

$a = 1, d = 0 \quad b = 1, c = 0$

$\therefore$ Total 10 cases when matrix is non invertible Total possible matrix $= 2^4 = 16$

Required probability of invertible

$= \frac{16-10}{16} = \frac{6}{16} = \frac{3}{8}$

Q2. Let $k_1 = 4\lambda_1 + r_1, r_1 \in \{0, 1, 2, 3\}$

(2) $k_2 = 4\lambda_2 + r_2$

$(i)^{k_1} + (i)^{k_2} = (i)^{r_1} + (i)^{r_2}$

$(i)^{r_1} \in \{1, i, -1, -i\}$

Zero $\Rightarrow 1, (-1)$ pair $\quad \Rightarrow \left\{\begin{matrix} 1, & -1 \\ i, & -i \\ -i, +i & \\ -1, & 1 \end{matrix}\right\}$

$i, (-i)$ pair

Zero probability $= \dfrac{4}{{}^4C_1 \cdot {}^4C_1} = \dfrac{1}{4}$

Probability (non-zero) $= 1 - \dfrac{1}{4} = \dfrac{3}{4}$

Q3. A, E,G R D N

(1) Probabllity (P) $= \frac{\text{favourable case}}{\text{Total case}}$

(when A \& E are in order)

Total case $= 6\,!$

Favourable case $= {}^6\mathrm{C}_2.4\,!$

$P = \frac{(15)4!}{(30)4!}$

Probablity when not in order $= 1 - \frac{1}{2} = \frac{1}{2}$

Q4. Bag contains 4 white and 6 black balls

(2) A : first ball selected is black

B : Second ball is also black

$$P\left(\frac{A}{B}\right)=\frac{\frac{6}{10}\times\frac{5}{9}}{\frac{4}{10}\times\frac{6}{9}+\frac{6}{10}\times\frac{5}{9}}=\frac{30}{24+30}$$

$$=\frac{30}{54}=\frac{5}{9}$$

$m+n=5+9=14$

Q5.
(4)

	x_i	P_i
HHH	0	$\frac{1}{8}$
TTT	0	$\frac{1}{8}$
HHT	1	$\frac{1}{8}$
HTH	1	$\frac{1}{8}$
THH	0	$\frac{1}{8}$
TTH	0	$\frac{1}{8}$
THT	1	$\frac{1}{8}$
HTT	1	$\frac{1}{8}$

$$\mu=\sum x_iP_i=\frac{1}{2}$$

$$\sigma^2=\sum x_i^2P_i-\mu^2$$

$$=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

$$64\left(\mu+\sigma^2\right)=64\left[\frac{1}{2}+\frac{1}{4}\right]$$

$$=64\times\frac{3}{4}=48$$

Q6. a = number or dice 1
(2) b = number on dice 2

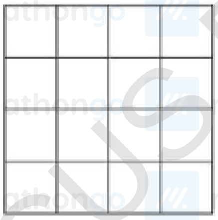
$(a,b)=(1,3),(3,1),(2,2),(2,3),(3,2),(1,4),(4,1)$

Required probability

$$=\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{2}{6}$$

$$=\frac{18}{36}=\frac{1}{2}$$

Q7.
(2)



Total $= {}^{16}\mathrm{C}_2$
Required ways $=$ Total $-$ (adjacent square)
$= {}^{16}\mathrm{C}_2 -$
[3 pair in vertical \& horizontal for
each row and column]
$= {}^{16}C_2 - [3 \times 4 + 3 \times 4]$
$= 96$
Probability $= \dfrac{96}{120} = \dfrac{4}{5}$

Q8. For sum ' 5 ' $\rightarrow (1,4),(2,3),(3,2)$
(2) $(4,1) \Rightarrow P(A) = \frac{4}{36}$
For sum ' 8 ' $\rightarrow (2,6),(3,5),(4,4)$
For sum ' 5 ' $\rightarrow (1,4),(2,3),(3,2)$
$(4,1) \Rightarrow P(A) = \frac{4}{36}$
For sum ' 8 ' $\rightarrow (2,6),(3,5),(4,4)$
$(5,3),(6,2) \Rightarrow P(B) = \dfrac{5}{36}$
$P(\bar{A}) = \dfrac{32}{36}, P(\bar{B}) = \dfrac{31}{36}$
$$P(A \text{ wins }) = P(A) + P(\bar{A})P(\bar{B})P(A) + + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \ldots$$
$$= \frac{P(A)}{1 - P(\bar{A})P(\bar{B})} = \frac{9}{19}$$

Q9. $P(A \cap B) = 0.1, P(A \mid B)$ and $P(B \mid A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$
(4)
$$\Rightarrow \quad P(A \mid B)P(B \mid A) = \frac{1}{12}$$
$$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} \times \frac{P(A \cap B)}{P(A)} = \frac{1}{12}$$
$$\Rightarrow \quad P(A)P(B) = 12(0.1)^2$$
$= 0.12$

Also, $P(A \mid B)+P(B \mid A)=\frac{7}{12}$

$$\Rightarrow P(A \cap B)\left(\frac{1}{P(B)}+\frac{1}{P(A)}\right)=\frac{7}{12}$$

$$\Rightarrow P(A)+P(B)=\frac{7}{12} \times \frac{0.12}{0.1}$$

$$\Rightarrow P(A)+P(B)=0.7$$

$$\frac{P(\bar{A} \cup \bar{B})}{P(\bar{A} \cap \bar{B})}=\frac{P(\overline{A \cap B})}{P(\overline{A U B})}$$

$$=\frac{1-P(A \cap B)}{1-P(A \cup B)}$$

$$=\frac{1-0.1}{1-(0.7-0.1)}=\frac{0.9}{0.4}=\frac{9}{4}$$

Q10. Bag 1 $\rightarrow 4w, 5B$

(1) Bag 2 $\rightarrow nw, 3B$

(I) $\rightarrow$ Transferred ball is white $P(w)=\frac{n+1}{n+4} \cdot \frac{4}{9}$

(II) $\rightarrow$ Transferred ball is black $P(w)=\frac{5}{9} \cdot \frac{n}{n+4}$

$$\frac{4n+4}{9n+36}+\frac{5n}{9n+36}=\frac{29}{45}$$

$$\frac{9n+4}{9n+36}=\frac{29}{45} \Rightarrow n=6$$

Q11. E_1 : $Bag B_1$ is selected

(1)

B_1	B_2	B_3
6W4B	4 W6 B	5 W5 B

E_2 : bag B_2 is selected

E_3 : $Bag B_3$ is selected

A : Drawn ball is white

We have to find $P\left(\frac{E_2}{A}\right)$

$$P\left(\frac{E_2}{A}\right)=\frac{P(E_2) P\left(\frac{A}{E_2}\right)}{P(E_1) P\left(\frac{A}{E_1}\right)+P(E_2) P\left(\frac{A}{E_2}\right)+P(E_3) P\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{3} \times \frac{4}{10}}{\frac{1}{3} \times \frac{6}{10}+\frac{1}{3} \times \frac{4}{10}+\frac{1}{3} \times \frac{5}{10}}$$

$$=\frac{4}{15}$$

Q12.
(3)

x	0	1	-1
$P(x)$	$\frac{10}{16}$	$\frac{3}{16}$	$\frac{3}{16}$

$$\operatorname{Var}(x)=E\left(x^2\right)-[E(x)]^2$$
$$=\sum_{i=1}^{3} x_i^2 P\left(x_i\right)-(\mu)^2$$
$$=1\times\frac{3}{16}+1\times\frac{3}{16} \quad [\mu=0]$$
$$=\frac{6}{16}=\frac{3}{8}$$

Q13. There are 3 bad oranges and 7 good oranges. $\therefore \quad X=$ number of bad oranges drawn.
(1)

X	0	1	2
$P(X)$	$\frac{{}^7C_2}{{}^{10}C_2}$	$\frac{{}^3C_1\times{}^7C_1}{{}^{10}C_2}$	$\frac{{}^3C_2}{{}^{10}C_2}$

$\therefore$ Variance
$$=0^2\cdot\frac{{}^7C_2}{{}^{10}C_2}+1^2\cdot\left(\frac{3\times 7}{{}^{10}C_2}\right)+2^2\left(\frac{3}{{}^{10}C_2}\right)$$
$$-\left(0+1\cdot\frac{3\times 7}{{}^{10}C_2}+2\cdot\frac{3}{{}^{10}C_2}\right)^2$$
$$=\frac{28}{75}$$

Q1. Let R be the required relation
(2) $\mathrm{A}=\{(1,1)(2,2),(3,3)\}$
(i) $|\mathrm{R}|=3$, when $\mathrm{R}=\mathrm{A}$
(ii) $|\mathrm{R}|=5$, e.g. $\mathrm{R}=\mathrm{A}\cup\{(1,2),(2,1)\}$
Number of R can be [3]
(iii) $\mathrm{R}=\{1,2,3\}\times\{1,2,3\}$
Ans. (5)

Q2. $A=\{1,2,3,4\}$
(2) For relation to be reflexive
$\mathrm{R}=\{(1,2),(2,3),(3,3)\}$
Minimum elements added will be
$(1,1),(2,2),(4,4)(2,1)(3,2)(3,2)(3,1)(1,3)$
$\therefore$ Minimum number of elements $=7$

Q3. $\sec^2 x-\tan^2 x=1$ (on replacing y with x)
(2) $\Rightarrow$ Reflexive
$\sec^2 x-\tan^2 y=1$
$\Rightarrow 1+\tan^2 x+1-\sec^2 y=1$
$\Rightarrow \sec^2 y-\tan^2 x=1$
$\Rightarrow$ symmetric
$\sec^2 x-\tan^2 y=1$
$\sec^2 y-\tan^2 z=1$
Adding both
$\Rightarrow \sec^2 x-\tan^2 y+\sec^2 y-\tan^2 z=1+1$
$\sec^2 x+1-\tan^2 z=2$
$\sec^2 x-\tan^2 z=1$
$\Rightarrow$ Transitive
hence equivalence releation

Q4. $\mathrm{A}=\{1,2,\ldots,10\}$
(2) $\mathrm{B}\left\{\frac{\mathrm{m}}{\mathrm{n}}=\mathrm{m},\mathrm{n}\in\mathrm{A},\mathrm{m}<\mathrm{n},\gcd(\mathrm{m},\mathrm{n})=1\right\}$
$\mathrm{n}(\mathrm{B})$
$\mathrm{n}=2 \quad \left\{\frac{1}{2}\right\}$
$\mathrm{n}=3 \quad \left\{\frac{1}{3},\frac{2}{3}\right\}$
$\mathrm{n}=4 \quad \left\{\frac{1}{4},\frac{3}{4}\right\}$
$\mathrm{n}=5 \quad \left\{\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5}\right\}$

n = 6 $\left\{\frac{1}{6}, \frac{5}{6}\right\}$

n = 7 $\left\{\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}\right\}$

n = 8 $\left\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\right\}$

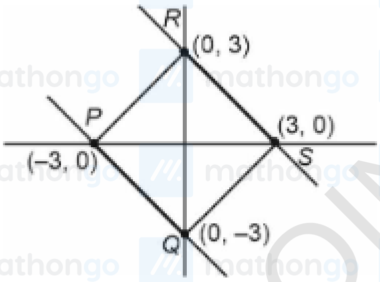
n = 9 $\left\{\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}\right\}$

n = 10 $\left\{\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}\right\}$

$n(B) = 31$

Q5. $A = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x + y| \geq 3\}$
(4) and $B = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x| + |y| \leq 3\}$

$C = \{(x, y) \in A \cap B : x = 0 \text{ or } y = 0\}$



$A \cap B$ will have only common points lying on the line PQ and RS

Now, $C = \{(-3, 0), (3, 0), (0, 3), (0, -3)\}$

$\sum_{(x,y) \in c} |x + y| = 3 + 3 + 3 + 3 = 12$

Q6. Let $\frac{y}{x} = \lambda$
(5120) $y = \lambda x$

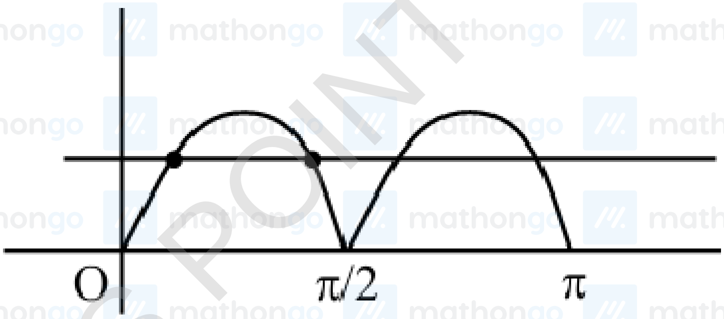
$= 10 \times \left({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + \ldots + {}^9C_9\right)$

$= 10 \times 2$

$= 10 \times 512 = 5120$

Q7. $\mathrm{A} : \log_{2\pi} |\sin x| + \log_{2\pi} |\cos x| = 2$
(2) $\Rightarrow \log_{2\pi}(|\sin x \cdot \cos x|) = 2$

$\Rightarrow |\sin 2x| = \frac{8}{\pi^2}$



Number of solution 4

B : let $\sqrt{x} = t < 2$

Then $\sqrt{x}(\sqrt{x}-4)+3(\sqrt{x}-2)+6=0$

$\Rightarrow \mathrm{t}^2-4\mathrm{t}+3\mathrm{t}-6+6=0$

$\Rightarrow \mathrm{t}^2-\mathrm{t}=0, \mathrm{t}=0, \mathrm{t}=1$

$\mathrm{x}=0, \mathrm{x}=1$

again let $\sqrt{\mathrm{x}}=\mathrm{t}>2$

then $\mathrm{t}^2-4\mathrm{t}-3\mathrm{t}+6+6=0$

$\Rightarrow \mathrm{t}^2-7\mathrm{t}+12=0$

$\Rightarrow \mathrm{t}=3,4$

$\mathrm{x}=9,16$

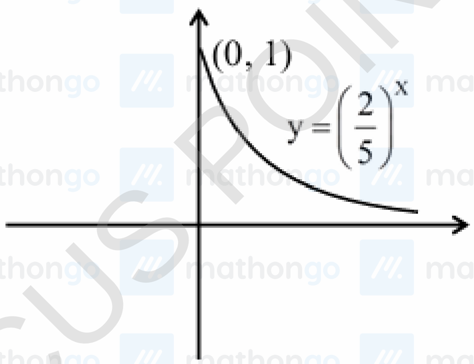
Total number of solutions

$\mathrm{n}(\mathrm{A} \cup \mathrm{B})=4+4=8$

Q8. $S=\{0,1,2,3 \ldots \ldots\}$

(4) $\log_y y=\log_e\left(\frac{2}{5}\right)$

$\Rightarrow y=\left(\frac{2}{5}\right)^x$



Required

Sum $=1+\left(\frac{2}{5}\right)^1+\left(\frac{2}{5}\right)^2+\left(\frac{2}{5}\right)^3+\ldots\ldots-=\frac{1}{1-\frac{2}{5}}=\frac{5}{3}$

Q9. R is reflexive $\Rightarrow R$ have $(1,1),(2,2),(3,3)$

(3) R is transitive

$\because (1,2),(23) \in R \quad \therefore (1,3) \in R$

$\therefore \quad R_1=\{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}$

Clearly R_1 is reflexive and transitive but not symmetric.

Similarly,

$R_2=\{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3),(3,2)\}$

$R_3=\{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3),(2,1)\}$

Therefore, 3 relations are possible

Q10. Reflexive : $(a_1, b) R (a_1, b_1) \Rightarrow b_1=b_1$ True

(2) Symmetric : $\left.\begin{array}{l}(a_1, b_1) R (a_2, b_2) \Rightarrow b_1=b_2 \\ (a_2, b_2) R (a_1, b_1) \Rightarrow b_2=b_1\end{array}\right\}$ True

Transitive : $(a_1, b_1) R (a_2, b_2) \Rightarrow b_1 = b_2$
$\& (a_2, b_2) R (a_3, b_3) b_2 = b_3$
$\Rightarrow (a_1, b_1) R (a_3 . b_3) \Rightarrow$ True $\} b_1 = b_3$

Hence Relation R is an equivence relation Statement-I is true.

For statement $-$II $\Rightarrow$ y $=$ b so False

Q11. For reflexive $(x, x) \in \mathbb{R}, x \in \mathbb{Z}$

(2) $\Rightarrow x + x = 2x \rightarrow$ even

For symmetric of $(x, y) \in \mathbb{R}$ then $(y, x) \in \mathbb{R}$ when $x, y \in \mathbb{Z}$

$x + y \rightarrow$ even

$\Rightarrow y + x \rightarrow$ even

for transitive if $(x, y) \in \mathbb{R} \Rightarrow x + y \rightarrow$ even

$(y, z) \in \mathbb{R} \Rightarrow y + z \rightarrow$ even

$x + 2y + z \rightarrow$ even

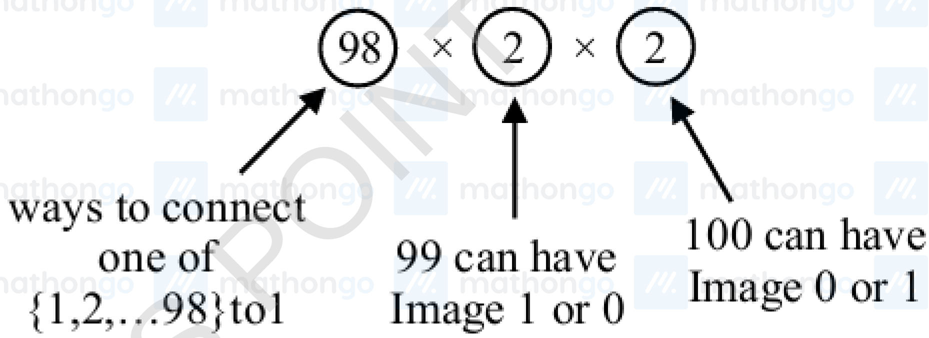
$\Rightarrow x + z$ is even

$\Rightarrow (x, z) \in \mathbb{R}$

$\Rightarrow \mathbb{R}$ is an equivalence relation.

Q1. as $f(x)$ is a polynomial of degree two let it be
(2) $f(x) = ax^2 + bx + c \quad (a \neq 0)$
on satisfying given conditions we get
$C = 1 \& a = \pm 1$
hence $f(x) = 1 \pm x^2$
also range $\in (-\infty, 1]$ hence
$f(x) = 1 - x^2$
now $f(k) = -2k$
$1 - k^2 = -2k \rightarrow k^2 - 2k - 1 = 0$
let roots of this equation be $\alpha \& \beta$ then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= 4 - 2(-1) = 6$

Q2.
(392)

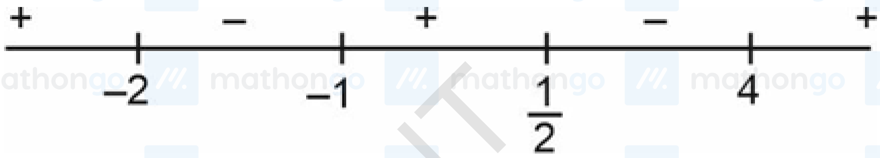


392 Ans.

Q3. $f(x) = \sec^{-1}(2[x] + 1)$
(3) $\Rightarrow 2[x] + 1 \geq 1$ or $2[x] + 1 \leq -1$
$\Rightarrow 2[x] \geq 0$ or $2[x] \leq -2$
$\Rightarrow [x] \geq 0$ or $[x] \leq -1$
$\Rightarrow x \geq 0$ or $x \leq 0$
Domain of $f(x)$ is $(-\infty, \infty)$

Q4. $f_1(x) = \log_5(18x - x^2 - 77)$
(3) $\therefore \quad 18x - x^2 - 77 > 0$
$x^2 - 18x + 77 < 0$
$x \in (7, 11)$
$\alpha = 7, \beta = 11$
$f_2(x) = \log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$
$x > 1, x - 1 \neq 1, \frac{2x^2+3x-2}{x^2-3x-4} > 0$
$x > 1, x \neq 2, \frac{(2x-1)(x+2)}{(x-4)(x+1)} > 0$

$x > 1, x \neq 2,$



$\therefore \quad x \in (4, \infty)$
$\therefore \quad \gamma = 4$
$\therefore \quad \alpha^2 + \beta^2 + \gamma^2 = 49 + 121 + 16$
$= 186$

Q5. $f(x) = 6 + 16\cos x \cdot \cos\left(\frac{\pi}{3} - x\right)$
(1)
$\cos\left(\frac{\pi}{3} + x\right) \cdot \sin 3x \cdot \cos 6x$
$f(x) = 6 + 4\cos 3x \cdot \sin 3x \cdot \cos 6x$
$\therefore \quad f(x) = 6 + \sin 12x$
$\therefore \quad$ Range of $f(x) = [5, 7]$
$\therefore \quad [\alpha, \beta] = [5, 7]$
$\therefore$ Distance of point from $3x + 4y + 12 = 0$
$= \left|\frac{3.5 + 4.7 + 12}{\sqrt{3^2 + 4^2}}\right|$
$= 11$ units

Q6. $f(g(x)) = \ln\left(\frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}\right)$
(4) Since $2x^2 - 2x + 1 > 0 \quad \forall x \in \mathbb{R} \because (-2)^2 - 4(2) < 0$
Consider
$x^4 - 2x^3 + 3x^2 - 2x + 2$
$= (x^4 - 2x^3 + x^2) + (x^2 - 2x + 1) + (1 + x^2)$
$= x^2(x-1)^2 + (x-1)^2 + (x^2+1) > 0 \forall x \in \mathbb{R}$
$\Rightarrow g(x) > 0 \forall x \in \mathbb{R}$
$\Rightarrow \ln f((x)), f(x) > 0 \forall x \in \mathbb{R}$
$\Rightarrow x \in \mathbb{R}$ is domain

Q7. $A = \{1, 2, 3, 4\}$
(1) $B = \{1, 4, 9, 16\}$

Total number of functions $= 4^4$

Total number of one-one functions $= 4!$

Total number of many one functions $= 4^4 - 4! = 232$

Total number of many-one functions in which $1 \notin f(A) = 3 \times 3 \times 3 \times 3 = 81$

$\therefore$ Total number of many one functions $1 \notin f(A)$
$= 232 - 81$
$= 151$

Q8. as $f(x)$ is onto hence A is range of $f(x)$
(2) $f'(x) = 6x^2 - 30x + 36$
$= 6(x-2)(x-3)$

now $f(2) = 16 - 60 + 72 + 7 = 35$
$f(3) = 54 - 135 + 108 + 7 = 34$
$f(0) = 7$

hence range $\in [7, 35] = A$

also for range of $g(x)$

$$g(x) = 1 - \frac{1}{x^{2025} + 1} \in [0, 1) = B$$

$s = \{0, 7, 8, \ldots .35\}$ hence $n(s) = 30$

Q9. $$f(x) = \frac{42^x + 16}{2.2^{2x} + 16.2^x + 32}$$
(2)
$$f(x) = \frac{2\left(2^x + 4\right)}{2^{2x} + 8.2^x + 16}$$
$$f(x) = \frac{2}{2^x + 4}$$
$$f(4 - x) = \frac{2^x}{2\left(2^x + 4\right)}$$
$$f(x) + f(4 - x) = \frac{1}{2}$$

So, $f\left(\frac{1}{15}\right) + f\left(\frac{59}{15}\right) = \frac{1}{2}$

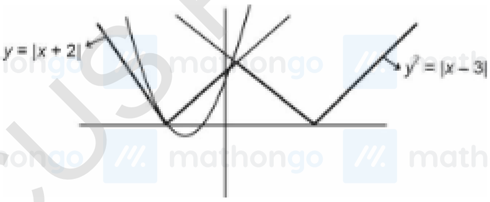
Similarly $= f\left(\frac{29}{15}\right) + f\left(\frac{31}{15}\right) = \frac{1}{2}$

$$f\left(\frac{30}{15}\right) = f(2) = \frac{2}{2^2 + 4} = \frac{2}{8} = \frac{1}{4}$$
$$\Rightarrow 8\left(29 \times \frac{1}{2} + \frac{1}{4}\right)$$

Ans. 118

Q10. $x^2+3x+2=\min\{|x-3|,|x+2|\}$

(3)



$$y=x^2+3x+2$$
$$y=x^2+2\left(\frac{3}{2}\right)x+\frac{9}{4}-\frac{9}{4}+2$$
$$y=\left(x+\frac{3}{2}\right)^2-\frac{1}{4}$$
$$y+\frac{1}{4}=\left(x+\frac{3}{2}\right)^2$$
$\Rightarrow$ Parabola vertex $\left(\frac{-3}{2},\frac{-1}{4}\right)$

$\Rightarrow$ By graph 2 solution possible

Q11. $f(x)-6f\left(\frac{1}{x}\right)=\frac{35}{3x}-\frac{5}{2}\ldots(1)$

(3)

$$6\left(f\left(\frac{1}{x}\right)-6f(x)=\frac{35x}{3}-\frac{5}{2}\right)$$
$$6f\left(\frac{1}{x}\right)-36f(x)=\frac{210x}{3}-\frac{30}{2}\ldots(2)$$
$(1)+(2)$
$$-35f(x)=\frac{35}{3}\left[\frac{1}{x}+6x\right]-\frac{5}{2}(1+6)$$
$$-f(x)=\frac{1}{3}\left(\frac{1}{x}+6x\right)-\frac{1}{2}$$
$$f(x)=-\frac{1}{3x}-2x+\frac{1}{2}$$
$$\lim_{x\to 0}\left[\frac{1}{\alpha x}-\frac{1}{3x}-2x+\frac{1}{2}\right]=\beta$$
$\Rightarrow \alpha=3$
$$\beta+2\beta=3+2\times\frac{1}{2}=4$$

Q12. $f(x)=\dfrac{2^x}{2^x+\sqrt{2}}$
(4)

$f(x)+f(1-x)=\dfrac{2^x}{2^x+\sqrt{2}}+\dfrac{2^{1-x}}{2^{1-x}+\sqrt{2}}$

$=\dfrac{2^x}{2^x+\sqrt{2}}+\dfrac{2}{2+\sqrt{2}2^x}=\dfrac{2^x+\sqrt{2}}{2^x+\sqrt{2}}=1$

Now, $\sum_{\mathrm{k}=1}^{81}\mathrm{f}\left(\frac{\mathrm{k}}{82}\right)=\mathrm{f}\left(\frac{1}{82}\right)+\mathrm{f}\left(\frac{2}{82}\right)+\ldots\ldots+\mathrm{f}\left(\frac{81}{82}\right)$

$=\mathrm{f}\left(\frac{1}{82}\right)+\mathrm{f}\left(\frac{1}{82}\right)+\ldots\ldots+\mathrm{f}\left(1-\frac{2}{82}\right)+\mathrm{f}\left(1-\frac{1}{82}\right)$

$\left[\mathrm{f}\left(\frac{1}{82}\right)+\mathrm{f}\left(1-\frac{1}{82}\right)\right]+\left[\mathrm{f}\left(\frac{2}{82}\right)+\mathrm{f}\left(1-\frac{2}{82}\right)\right]+\ldots .40\text{ cases }+\mathrm{f}\left(\frac{41}{82}\right)$

$(1+1+\ldots .+1)40\text{ times }+\dfrac{2^{1/2}}{2^{1/2}+2^{1/2}}$

$40+\dfrac{1}{2}=\dfrac{81}{2}$

Q13. $\mathrm{f}(\mathrm{x})=\dfrac{2^{2\mathrm{x}}-1}{2^{2\mathrm{x}}+1}$
(4)

$=1-\dfrac{2}{2^{2\mathrm{x}}+1}$

$\mathrm{f}'(\mathrm{x})=\dfrac{2}{\left(2^{2\mathrm{x}}+1\right)^2}\cdot 2.2^{2\mathrm{x}}\cdot\ln 2$ i.e always $+$ve

so $f(x)$ is $\uparrow$ function

$\therefore \mathrm{f}(-\infty)=-1$

$\mathrm{f}(\infty)=1$

$\therefore \mathrm{f}(\mathrm{x})\in(-1,1)\neq$ co-domain

so function is one-one but not onto

Q14. Put $y=0$
(4)

$f(x)=f(0)+f(x)+1-0$

$f(0)=-1$

$f(0)=0+0+b$

$\Rightarrow b=-1$

$f(-1+1)=f(-1)+f(1)+1+\dfrac{2}{7}$

$f(0)=f(-1)+f(1)+\dfrac{9}{7}$

$-1=(2+3a)+\left(\dfrac{a+2}{a-1}\right)(-1)+b+(2+3a)$

$+\dfrac{a+2}{a-1}+b+\dfrac{9}{7}$

$$-1 = 4 + 6a - 2 + \frac{9}{7}$$

$$-1 = 2 + \frac{9}{7} + 6a$$

$$6a = -1 - 2 - \frac{9}{7}$$

$$a = \frac{-5}{7}$$

$$f(x) = \frac{-x^2}{7} + \frac{\frac{9}{7}}{\frac{-12}{7}}x - 1$$

$$f(x) = \frac{-x^2}{7} - \frac{3}{4}x - 1$$

$$\sum_{i=1}^{5} f(i) = -\frac{1}{7}\left(\frac{5 \times 6 \times 11}{6}\right) - \frac{3}{4}\left(\frac{5 \times 6}{2}\right) - 5$$

$$= \frac{-55}{7} - \frac{45}{4} - 5$$

$$= \frac{675}{28}$$

$$\Rightarrow 28\left|\sum_{i-1}^{5} f(i)\right| = 675$$

Q1. $\lim\limits_{x \to 0} \operatorname{cosec}\left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$

(4)

$$\lim_{x\to 0} \frac{\cos \text{ec}\, x\left(\cos^2 x + 3\cos x - \sin x - 4\right)}{\left(\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}\right)}$$

$$\lim_{x\to 0} \frac{1}{\sin x} \frac{\left(\cos^2 x + 3\cos x - 4\right) - \sin x}{\left(\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}\right)}$$

$$\lim_{x\to 0} \frac{\left(\cos^x + 4\right)\left(\cos x - 1\right) - \sin x}{\sin x\left(\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}\right)}$$

$$\lim_{x\to 0} \frac{-2\sin^2 \frac{x}{2}(\cos x + 4) - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}\left(\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}\right)}$$

$$\lim_{x\to 0} \frac{-\left(\sin^x(\cos x + 4) + \cos\frac{x}{2}\right)}{\cos\frac{x}{2}\left(\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}\right)}$$

$$-\frac{1}{2\sqrt{5}}$$

Q2. $\alpha = \lim\limits_{x\to\infty}\left(\left(\frac{\mathrm{e}}{1-\mathrm{e}}\right)\left(\frac{1}{\mathrm{e}} - \frac{\mathrm{x}}{1+\mathrm{x}}\right)\right)^{\mathrm{x}}$ (1^{∞} form)

(4)

$\therefore \alpha = \mathrm{e}^{\mathrm{L}}$

Where $\mathrm{L} = \lim\limits_{x\to\infty} \mathrm{x}\left(\left(\frac{\mathrm{e}}{1-\mathrm{e}}\right)\left(\frac{1}{\mathrm{e}} - \frac{\mathrm{x}}{1+\mathrm{x}}\right) - 1\right)$

$\Rightarrow \mathrm{L} = \lim\limits_{\mathrm{x}\to\infty}\left(\frac{\mathrm{e}}{1-\mathrm{e}}\right)\mathrm{x}\left(\frac{1}{\mathrm{e}} - \frac{\mathrm{x}}{1+\mathrm{x}} - \left(\frac{1-\mathrm{e}}{\mathrm{e}}\right)\right)$

$\Rightarrow \mathrm{L} = \frac{\mathrm{e}}{1-\mathrm{e}} \lim\limits_{\mathrm{x}\to\infty} \mathrm{x}\left(1 - \frac{\mathrm{x}}{1+\mathrm{x}}\right)$

$\Rightarrow \mathrm{L} = \frac{\mathrm{e}}{1-\mathrm{e}} \lim\limits_{\mathrm{x}\to\infty} \frac{\mathrm{x}}{\mathrm{x}+1}$

$\Rightarrow \mathrm{L} = \frac{\mathrm{e}}{1-\mathrm{e}} \cdot 1$

$\Rightarrow \mathrm{L} = \frac{\mathrm{e}}{1-\mathrm{e}}$

$\therefore \alpha = \mathrm{e}^{\frac{\mathrm{e}}{1-\mathrm{e}}} \Rightarrow \log\alpha = \frac{\mathrm{e}}{1-\mathrm{e}}$

$\therefore$ Required value $= \frac{\frac{\mathrm{e}}{1-\mathrm{e}}}{1 + \frac{\mathrm{e}}{1-\mathrm{e}}} = \mathrm{e}$

Q3.
(64)

$$\frac{\alpha}{5e} = \exp\left(\lim_{t\to 0}\frac{1}{t}\left(\int_0^1 (3x+5)^t dx - 1\right)\right)$$

$$= \exp\left(\lim_{t\to 0}\frac{1}{t}\left(\frac{(3x+5)^{t+1}}{3(t+1)}\right)_0^1 - 1\right)$$

$$= \exp\left(\lim_{t\to 0}\frac{1}{t}\left(\frac{8^{t+1}-5^{t+1}}{3(t+1)} - 1\right)\right)$$

$$= \exp\left(\lim_{t\to 0}\frac{1}{t}\left(\frac{8^{t+1}-5^{t+1}-3t-3}{3(t+1)}\right)\right)$$

$$= \exp\left(\lim_{t\to 0}\left(\frac{8^{t+1}\cdot\ln 8-5^{t+1}\ln 5-3}{3(t+1)}\right)\right)$$

$$= \exp\left(\frac{\ln 8^8-\ln 5^5-3}{5}\right)$$

$$= \left(\frac{8}{5}\right)^{2/3}\frac{\alpha}{5e} = \exp\left(\frac{\ln\left(\frac{8^8}{5^5}\right)}{5} - 1\right)$$

$$\Rightarrow \left(\frac{8}{5}\right)^{2/3}\frac{\alpha}{5} = \left(\frac{8^8}{5^5}\right)^{1/3} = \left(\frac{8^6\cdot 8^2}{5^3\cdot 5^2}\right)^{1/3} = \frac{64}{5}\left(\frac{8}{5}\right)^{2/3}$$

$$\Rightarrow \alpha = 64$$

Q4.
(24)

$$\lim_{x\to 0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\left[\frac{9^2}{x^2}\right]\right)\right)\geq 1$$

$$(1+2+\ldots\ldots+p)-\left(1^2+2^2+\ldots 9^2\right)\geq 1$$

$$\frac{p(p+1)}{2}-\frac{9.10.19}{6}\geq 1$$

$$p(p+1)\geq 572$$

Least natural value of p is 24

Q5.
(3)

$$\lim_{x\to\infty}\frac{\left(2x^2-3x+5\right)(3x-1)^{x/2}}{\left(3x^2+5x+4\right)\sqrt{(3x+2)^x}}$$

$$= \lim_{x\to\infty}\frac{x^2\left(2-\frac{3}{x}+\frac{5}{x^2}\right)(3x)^{\frac{x}{2}}}{x^2\left(3+\frac{5}{x}+\frac{4}{x^2}\right)(3x)^{\frac{x}{2}}}\cdot\frac{\left(1-\frac{1}{3x}\right)^{\frac{x}{2}}}{\left(1+\frac{2}{3x}\right)^{\frac{x}{2}}}$$

$$\lim_{x\to\infty}\left(1-\frac{1}{3x}\right)^{\frac{x}{2}} = e^{\lim_{x\to\infty}\left(1-\frac{1}{3x}-1\right)\times\frac{x}{2}} = e^{\frac{-1}{6}}$$

$$\lim_{x\to\infty}\left(1+\frac{2}{3x}\right) = e^{\lim_{x\to\infty}\left(1+\frac{2}{3x}-1\right)\times\frac{x}{2}}$$

$$= e^{\frac{1}{3}}$$

So, $\frac{2}{3}\times\frac{e^{\frac{-1}{6}}}{e^{\frac{1}{3}}} = \frac{2}{3}\times\frac{1}{e^{\frac{1}{3}+\frac{1}{6}}} = \frac{2}{3\sqrt{e}}$

Q6.
(1)

$$f(x)=\lim_{x\to\infty}\left(\frac{\tan\left(x/2^{r+1}\right)+\tan^3\left(x/2^{r+1}\right)}{1-\tan^2\left(x/2^{r+1}\right)}\right)$$

$$=\lim_{x\to\infty}\frac{\tan\left(\frac{x}{2^{r+1}}\right)}{\cos\left(\frac{x}{2^r}\right)}$$

$$=\lim_{x\to\infty}\frac{\sin\left(x/2^{r+1}\right)}{\cos\left(\frac{x}{2^{r+1}}\right)\cos\left(\frac{x}{2^r}\right)}$$

$$=\lim_{x\to\infty}\frac{\sin\left(\frac{x}{2^r}-\frac{x}{2^{r+1}}\right)}{\cos\left(\frac{x}{2^{r+1}}\right)\cos\left(\frac{x}{2^r}\right)}$$

$$=\lim_{x\to\infty}\tan\left(\frac{x}{2^r}\right)-\tan\left(\frac{x}{2^{r+1}}\right)$$

From condition given question

$$\therefore\lim_{n\to\infty}\sum_{r=0}^{n}\left[\tan\left(\frac{x}{2^r}\right)-\tan\left(\frac{x}{2^{r+1}}\right)\right]=\tan x$$

$$\therefore\lim_{x\to 0}\left(\frac{e^x-e^{\tan x}}{x-\tan x}\right)$$

$$\Rightarrow\lim_{x\to 0}e^{\tan x}\left(\frac{e^{x-\tan x}-1}{x-\tan x}\right)$$

$$\Rightarrow\lim_{x\to 0}e^{\tan x}\lim_{x\to 0}\left(\frac{e^{x-\tan x}-1}{x-\tan x}\right)$$

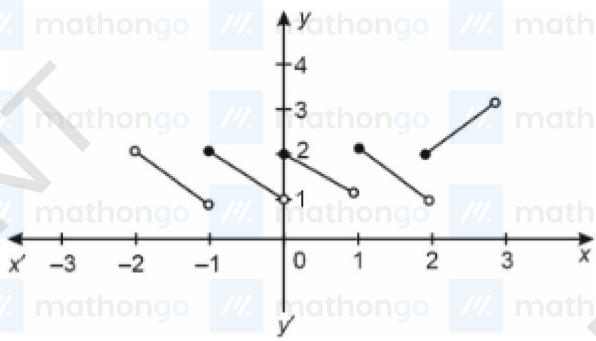
$$\Rightarrow 1.1\left(\because\lim_{x\to 0}\frac{e^{x-1}}{x}=1\right)$$

$$=1$$

Q1. $\lim_{x\to0^-}\frac{2}{x}\{\sin(k_1+1)x+\sin(k_2-1)x\}=4$
(4) $\Rightarrow 2(k_1+1)+2(k_2-1)=4$
$\Rightarrow k_1+k_2=2$
$\Rightarrow \lim_{x\to0^+}\frac{2}{x}\ln\left(\frac{2+k_1x}{2+k_2x}\right)=4$
$\Rightarrow \lim_{x\to0^+}\frac{1}{x}\ln\left(1+\frac{(k_1-k_2)x}{2+k_2x}\right)=2$
$\Rightarrow \frac{k_1-k_2}{2}=2$
$\Rightarrow k_1-k_2=4$
$\therefore k_1=3, k_2=-1$
$k_1^2+k_2^2=9+1=10$

Q2. $\because f(x+y)=f(x)\cdot f(y)+f(x)\cdot f(y), \forall x,y\in R$ ....(i)
(1) And $f(0)=1$ ....(ii)
Now replace x by zero and y by zero we get
$f(0)=f(0)f(0)+f(0)f(0)$
$1=f(0)+f(0)$
$\therefore \quad f'(0)=\frac{1}{2}\ldots(iii)$
Now replace y by zero in equation (i), we get
$f(x)=\frac{1}{2}f(x)+f'(x)$
or, $\frac{1}{2}f(x)=f'(x)$
then $\frac{f'(x)}{f(x)}=\frac{1}{2}$
hence $\ln|f(x)|=\frac{x}{2}+c$
Put $x=0$, we get $c=0$
$\therefore \quad \ln|f(x)|=\frac{x}{2}$
Then $\sum_{n=1}^{100}\ln(f(\eta))=\left(\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+\ldots+\frac{100}{2}\right)$
$=\frac{5050}{2}=2525$

Q3.
(2)



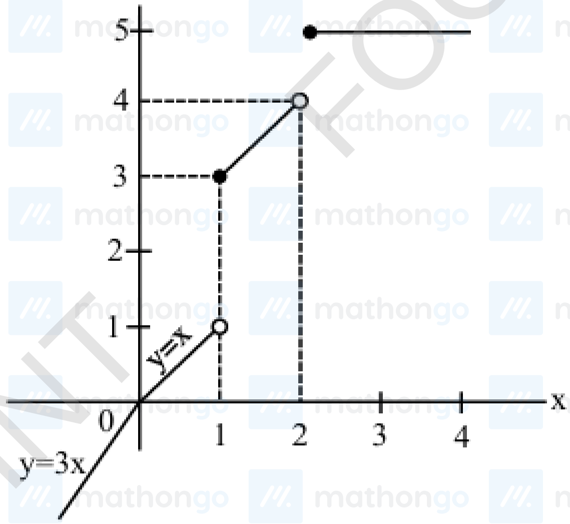
$f(x)=[x]+|x-2|, -2<x<3$

$$\therefore f(x)=\begin{cases}-x, -2<x<-1\\ 1-x, -1\le x<0\\ 2-x, 0\le x<1\\ 3-x, 1\le x<2\\ x, 2\le x<3\end{cases}$$

It is clearly discontinues at 4 points and nondifferentiable at 4 points.

$\therefore \quad m + n = 8$

Q4.
(5) $f(x) = \begin{cases} 3x & ; \quad x < 0 \\ \min\{1+x, x\} & ; \quad 0 \le x < 1 \\ \min\{2+x, x+2\} & ; \quad 1 \le x < 2 \\ 5 & ; \quad x > 2 \end{cases}$



$f(x) = \begin{cases} 3x & ; \quad x < 0 \\ x & ; \quad 0 \le x < 1 \\ x+2 & ; \quad 1 \le x < 2 \\ 5 & ; \quad x > 2 \end{cases}$

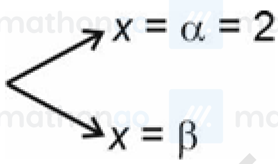
Not continuous at x $\in \{1, 2\} \Rightarrow \alpha = 2$

Not diff. at $x \in \{0, 1, 2\} \Rightarrow \beta = 3 \; \alpha + \beta = 5$

Q5. $f(x) = \left(x^2 + 1\right)\left|x^2 - ax + 2\right| + \cos|x|$

(3) Notice that $\cos(-x) = \cos x = \cos|x|$ which means $\cos|x|$ is differentiable everywhere in $x \in R$

$\Rightarrow f(x)$ can be non differentiable where $\left|x^2 - ax + 2\right|$ $= 0$



$\Rightarrow x^2 - ax + 2 = 0$

$\Rightarrow 4 - 2a + 2 = 0 \Rightarrow a = 3$

$\Rightarrow \quad \left(x^2 - 3x + 2\right) = 0 \quad \Rightarrow \quad x = 1, 2$

$\beta = 1$

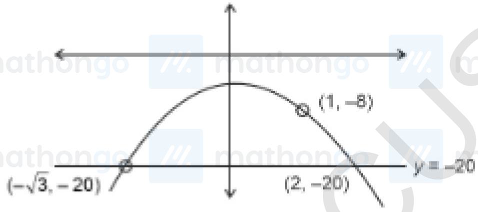
distance of (α, β) from line

$12x + 5y + 10 = 0$

$\Rightarrow \dfrac{|2(12) + 5(1) + 10|}{13} = \dfrac{39}{13} = 3$

Q6. $f(x)$ is continuous and differentiable

(34)



at $x=1, \mathrm{LHL}=\mathrm{RHL}, \mathrm{LHD}=\mathrm{RHD}$

$-3a-2=a^2+b, -6a=b$

$a=2; b=-12$

$$f(x)=\begin{cases}-6x^2-2, & x<1\\ 4-12x, & x\geq 1\end{cases}$$

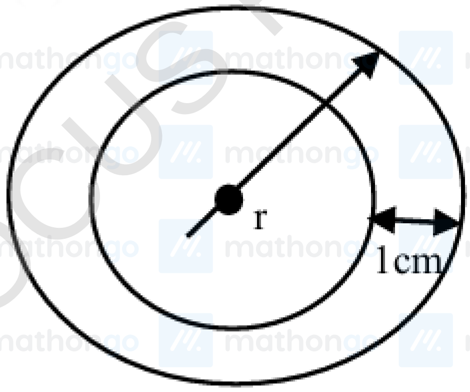
$$\text{Area } = \int_{-\sqrt{3}}^{1}\left(-6x^2-2+20\right)dx+\int_{1}^{2}(4-12x+20)dx\Big]$$

$=16+12\sqrt{3}+6=22+12\sqrt{3}$

$\therefore \quad \alpha+\beta=34$

FOCUS POINT mathongo

Q1.
(2)



$v = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$81 = 4\pi r^2 \times \frac{1}{4\pi}$

$r^2 = 81$

$r = 9$

surface area of chocolate $= 4\pi(r-1)^2 = 256\pi$

Q2.
(2)

$f'(x) = \frac{2}{x-2} - 2x + a \geq 0$

$f''(x) = \frac{-2}{(x-2)^2} - 2 < 0$

$f'(x) \downarrow$

$f'(3) \geq 0$

$2 - 6 + a \geq 0$

$a \geq 4$

$a_{min} = 4$

$g(x) = (x-1)^3(x+2-a)^2$

$g(x) = (x-1)^3(x-2)^2$

$g'(x) = (x-1)^3 2(x-2) + (x-2)^2 3(x-1)^2$

$= (x-1)^2(x-2)(2x-2+3x-6)$

$= (x-1)^2(x-2)(5x-8) < 0$

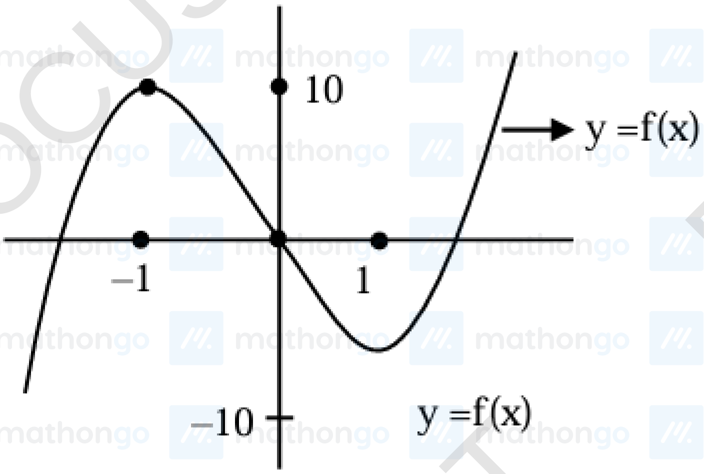
$x \in \left(\frac{8}{5}, 2\right)$

$100(a+b-c) = 100\left(4 + \frac{8}{5} - 2\right) = 360$

Q3. $5x^3 - 15x - a = 0$

(30) $f(x) = 5x^3 - 15x$

$f(x) = 15x^2 - 15 = 15(x-1)(x+1)$



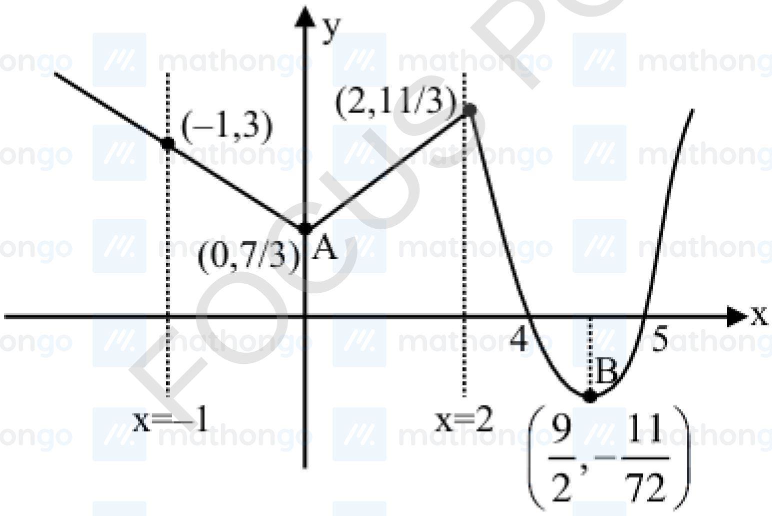
a $\in (-10, 10)$

$\alpha = -10, \beta = 10$

$\beta - 2\alpha = 10 + 20 = 30$

Q4.

(1)



$$f(x) = \begin{cases} 1 - 2x, & x < -1 \\ \frac{1}{3}(7 - 2x), & -1 \le x \le 2 \\ \frac{1}{3}(7 + 2x) & 0 \le x < 2 \\ \frac{11}{18}(x-4)(x-5), & x > 2 \end{cases}$$

$\therefore$ Local minimum values at A& B

$$\frac{7}{3} - \frac{11}{72}$$

$$\Rightarrow \frac{168 - 11}{72} \Rightarrow \frac{157}{72}$$

Q1.
(2)

$$I(x)=\int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$$

Put $\frac{x-11}{x+15}=t \Rightarrow \frac{26}{(x+5)^2}dx=dt$

$$I(x)=\frac{1}{26}\int \frac{dt}{t^{11/13}}=\frac{1}{26}\cdot\frac{t^{2/13}}{2/13}$$

$$\mathrm{I(x)}=\frac{1}{4}\left(\frac{\mathrm{x}-11}{\mathrm{x}+15}\right)^{2/13}+\mathrm{C}$$

$$\mathrm{I(37)}-\mathrm{I(24)}=\frac{1}{4}\left(\frac{26}{52}\right)^{2/13}-\frac{1}{4}\left(\frac{13}{39}\right)^{2/13}$$

$$=\frac{1}{4}\left(\frac{1}{2^{2/13}}-\frac{1}{3^{2/13}}\right)$$

$$=\frac{1}{4}\left(\frac{1}{4^{1/13}}-\frac{1}{9^{1/13}}\right)$$

$\therefore \mathrm{b}=4, \mathrm{c}=9$

$3(\mathrm{b}+\mathrm{c})=39$

Q2.
(16)

$2\mathrm{x}^2+5\mathrm{x}+9=\mathrm{A}\left(\mathrm{x}^2+\mathrm{x}+1\right)+\mathrm{B}(2\mathrm{x}+1)+\mathrm{C}$

$\mathrm{A}=2 \quad \mathrm{B}=\frac{3}{2} \quad \mathrm{C}=\frac{11}{2}$

$$2\int\sqrt{\mathrm{x}^2+\mathrm{x}+1}\,\mathrm{dx}+\frac{3}{2}\int\frac{2\mathrm{x}+1}{\sqrt{\mathrm{x}^2+\mathrm{x}+1}}\mathrm{dx}+\frac{11}{2}\int\frac{\mathrm{dx}}{\sqrt{\mathrm{x}^2+\mathrm{x}+1}}$$

$$2\int\sqrt{\left(\mathrm{x}+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,\mathrm{dx}+3\sqrt{\mathrm{x}^2+\mathrm{x}+1}+\frac{11}{2}\int\frac{\mathrm{dx}}{\sqrt{\left(\mathrm{x}+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$2\left(\frac{\mathrm{x}+\frac{1}{2}}{2}\sqrt{\mathrm{x}^2+\mathrm{x}+1}+\frac{3}{8}\ln\left(\mathrm{x}+\frac{1}{2}+\sqrt{\mathrm{x}^2+\mathrm{x}+1}\right)\right)+3\sqrt{\mathrm{x}^2+\mathrm{x}+1}$$

$$+\frac{11}{2}\ell\mathrm{n}\left(\mathrm{x}+\frac{1}{2}+\sqrt{\mathrm{x}^2+\mathrm{x}+1}\right)+\mathrm{C}$$

$\alpha=\frac{7}{2} \quad \beta=\frac{25}{4}$

$\alpha+2\beta=16$

Q3.
(2) $\frac{d}{dx}\left(\frac{x\cdot\sin^{-1}x}{\sqrt{1-x^2}}\right)-\sin^{-1}x\cdot\left(\frac{1\cdot\sqrt{1-x^2}+\frac{x\cdot 2x}{2\sqrt{1-x^2}}}{1-x^2}\right)$

$=\frac{x}{\sqrt{1-x^2}}\cdot\frac{1}{\sqrt{1-x^2}}$

$=\frac{\sin^{-1}x}{\left(1-x^2\right)^{3/2}}+\frac{x}{1-x^2}$

Hence, $I=\int e^x\left(f(x)+f'(x)\right)dx$

$=e^x\cdot f(x)+C$

$I=e^x\cdot\frac{x\cdot\sin^{-1}x}{\sqrt{1-x^2}}+C=g(x)+C$

$\Rightarrow g(x)=\frac{xe^x\sin^{-1}x}{\sqrt{1-x^2}}$ and $g(1/2)=\frac{\pi}{6}\sqrt{\frac{e}{3}}$

Q4.
(2) $\int x^3\sin x dx=-x^3\cos x+\int 3x^2\cos x dx$

$=-x^3\cos x+3x^2\sin x-\int 6x\sin x dx$

$=-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x+c$

So $g(x)=-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x$

$\mathrm{g}\left(\frac{\pi}{2}\right)=\frac{3\pi^2}{4}-6$

$\mathrm{g}'(\mathrm{x})=-3\mathrm{x}^2\cos\mathrm{x}+\mathrm{x}^3\sin\mathrm{x}+6\cos\mathrm{x}-6\cos\mathrm{x}$

$\mathrm{g}'\left(\frac{\pi}{2}\right)=\frac{\pi^3}{8}$

$8\left(\mathrm{g}\left(\frac{\pi}{2}\right)+\mathrm{g}'\left(\frac{\pi}{2}\right)\right)=\pi^3+6\pi^2-48$

So $\alpha+\beta-\gamma=55$

Q5. Put $x^{1/4}=t\Rightarrow dx=4t^3dt$
(1) $\int\frac{4t^3dt}{t(t+1)}=4\int\left(\frac{t^2-1}{t+1}+\frac{1}{t+1}\right)dt$

$f(x)=4\left[\frac{x^{1/2}}{2}-x^{1/4}+\ln\left|x^{1/4}+1\right|\right]+C$

$f(0)=-6$

$\Rightarrow C=-6$

$\Rightarrow\quad f(1)=4\left(\log_e 2-2\right)$

Q1. (2)

$$I=\int_0^{\frac{\pi}{4}}\left(\frac{\sin\theta+\cos\theta}{9-16\sin 2\theta}\right)d\theta$$

Take $\sin\theta-\cos\theta=t$

$(\cos\theta+\sin\theta)d\theta=dt$

$(\sin\theta-\cos\theta)^2=t^2$

$\Rightarrow \sin 2\theta=1-t^2$

$\theta=0\to t=-1$

$\theta=\frac{\pi}{4}\to t=0$

$$I=\int_{-1}^{0}\frac{dt}{9+16\left(1-t^2\right)}$$

$$=\frac{1}{16}\int_{-1}^{0}\frac{dt}{25}\frac{16}{16}-t^2$$

$$=\frac{1}{4}\left[\frac{1}{10}\log\frac{5-4t}{|\,5+4t}\right]_{-1}^{0}$$

$$=\frac{1}{40}\left[0+\log_e 9\right]$$

$$I=\frac{\log_e 9}{40}$$

$80I=2\log_e 9$

$80I=4\log_e 3$

Q2. (1)

$$\int_0^2 \mathrm{xF'(x)dx}=6$$

$$=\mathrm{xF(x)}\big|_0^2-\int_0^2 \mathrm{f(x)dx}=6$$

$$=2\,\mathrm{F}(2)-\int_0^2 \mathrm{xF(x)dx}=6[\because \mathrm{f}(2)=2\,\mathrm{F}(2)=2]$$

$$\int_0^2 \mathrm{xF(x)dx}=-2\ldots(1)$$

$$\Rightarrow\int_0^2 \mathrm{F(x)dx}=-2\ldots(2)$$

Also

$\int_0^2 x^2F''(x)dx=x^2F'(x)\big|_0^2-2\int_0^2 x'F'(x)dx=40$

$=4F'(2)-2\times 6=40$

$F'(2)=13$

$$\therefore F'(2)+\int_0^2 F(x)=13-2=11$$

Q3. (12)

Let $I=24\int_0^{\frac{\pi}{2}}\left(\sin\left|4x-\frac{\pi}{2}\right|+[2\sin x]\right)dx$ ...(i)

Now $\left|4x-\frac{\pi}{12}\right|=\begin{cases}-4x+\frac{\pi}{12} & ;x<\frac{\pi}{48}\\ 4x-\frac{\pi}{12} & ;\quad x\geq\frac{\pi}{48}\end{cases}$

$\therefore$ from (i)

$$I = 24\int_0^{\frac{\pi}{48}} -\sin\left(4x - \frac{\pi}{12}\right)dx + \int_{\frac{\pi}{48}}^{\frac{\pi}{4}} \sin\left(4x - \frac{\pi}{12}\right)$$

$$+\int_0^{\frac{\pi}{6}} [2\sin x]dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} [2\sin x]dx$$

$$I = 24\left[\frac{\left(1 - \cos\frac{\pi}{12}\right)}{4} - \frac{\left(-\cos\frac{\pi}{12} - 1\right)}{4}\right) + \frac{\pi}{4} - \frac{\pi}{6}$$

$$I = 24\left(\frac{1}{2}\right) + \frac{\pi}{4} - \frac{\pi}{6}$$

$I = 2\pi + 12 = 2\pi + \alpha$ (from above)

$\therefore \quad \alpha = 12$

Q4.
(1)

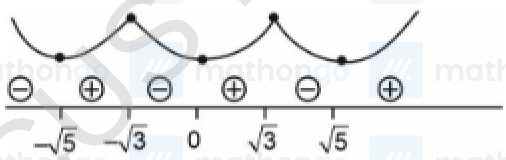
$$f(x) = \int_0^{x^2} \frac{t^2 - 8t + 15}{e^t} dt, x \in R$$

$$f'(x) = \frac{x^4 - 8x^2 + 15}{e^{x^2}}(2x) = 0$$

$$\Rightarrow \quad \frac{2 \times \left(x^2 - 5\right)\left(x^2 - 3\right)}{e^{x^2}} = 0$$

$\Rightarrow x(x + \sqrt{5})(x - \sqrt{5})(x + \sqrt{3})(x - \sqrt{3}) = 0$

By using wavy curve method



Number of local maximum $= 2$

Number of local minimum $= 3$

Q5.
(1)

$$\int_0^x tf(t)dt = x^2 f(x)$$

$\Rightarrow \quad xf(x) = 2xf(x) + x^2 f'(x)$

$\Rightarrow x^2 \cdot f'(x) = -xf(x)$

$$\Rightarrow \frac{f'(x)}{f(x)} = -\frac{1}{x}$$

$$\int \frac{f'(x)}{f(x)dx} = -\int \frac{1}{x}dx$$

$\Rightarrow \quad \ln f(x) = -\ln x + C$

$f(2) = 3 \Rightarrow \ln 3 = -\ln 2 + C$

$C = \ln 6$

$$\Rightarrow \quad f(x) = \frac{6}{x} \Rightarrow f(6) = 1$$

Q6. $g(x)=x^2+x^{7/3}$

(4) $g'(x)=2x+\frac{7}{3}x^{4/3}$

$f(x)=\frac{g'(x)}{x}$

$f(x)=2+\frac{7}{3}x^{1/3}$

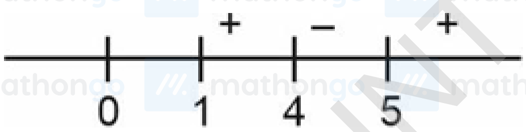
$f\left(r^3\right)=2+\frac{7}{3}r$

$\sum_{r=1}^{15}\left(2+\frac{7}{3}r\right)=2(15)+\frac{7}{3}\left(\frac{15(16)}{2}\right)$

$=310$

Q7. $f'(x)=x\left(x^2-9x+20\right), x\in(1,5)$

(4) $=(x-4)x(x-5)$



$\Rightarrow f'(x)>0\forall x\in(1,4)$
$\Rightarrow f'(x)<0\forall x\in(4,5)$
$\Rightarrow f(x)$ increasing in $(1,4)$
$f(x)$ decreasing in $(4,5)$
$\Rightarrow$ critical points to check:
$x=1,4,5$

$f(x)=\int_0^x\left(t^3-9t^2+20t\right)dt$

$=\frac{t^4}{4}-3t^3+10t^2\Big|_0^x=\frac{x^4}{4}-3x^3+10x^2$

$f(1)=\frac{1}{4}-3+10=\frac{29}{4}$

$f(4)=4^3-3.4^3+10.4^2=-2.4^3+10.4^2=32$

$f(5)=\frac{5^4}{4}-3.5^3+10.25=\frac{5^4}{4}-125=\frac{125}{4}$

Range $\Rightarrow\left[\frac{29}{4},32\right]\Rightarrow 4(\alpha+\beta)=128+29$

$=157$

Q8. (4)

$$I(m, m)=\int_0^1 x^{m-1}(1-x)^{n-1} dx$$

Let $x=\sin^2\theta \quad dx=2\sin\theta\cos\theta\, d\theta$

$$I(m, n)=2\int_0^{\pi/2}(\sin\theta)^{2m-1}(\cos\theta)^{2n-1} d\theta$$

$$I(9,14)+I(10,13)=2\int_0^{\pi/2}(\sin\theta)^{17}(\cos\theta)^{27} d\theta$$

$$+2\int_0^{\pi/2}(\sin\theta)^{19}(\cos\theta)^{25} d\theta$$

$$=2\int_0^{\pi/2}(\sin\theta)^{17}(\cos\theta)^{25} d\theta$$

$$=I(9,13)$$

Q9. (2)

$$f(x)=7\tan^8 x+7\tan^6 x-3\tan^4 x-3\tan^2 x$$

$$=7\tan^6 x\left(1+\tan^2 x\right)-3\tan^2 x\left(1+\tan^2 x\right)$$

$$=\left(7\tan^6 x-3\tan^2 x\right)\left(1+\tan^2 x\right)$$

$$=\left(7\tan^6 x-3\tan^2 x\right)\sec^2 x$$

$$l_1=\int_0^{\frac{\pi}{4}} f(x) dx=\int_0^{\frac{\pi}{4}}\left(7\tan^6 x-3\tan^2 x\right)\sec^2 x dx$$

$$=\left.\left(\frac{7\tan^7 x}{7}-\frac{3\tan^3 x}{3}\right)\right|_0^{\frac{\pi}{4}}=1-1=0$$

$$I_2=\int_0^{\frac{\pi}{4}} x f(x) dx=\int_0^{\frac{\pi}{4}} x\left(7\tan^6 x-3\tan^2 x\right)\sec^2 x dx$$

$$=\left.x\left(\tan^7 x-\tan^3 x\right)\right|_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}} 1\cdot\left(\tan^7 x-\tan^3 x\right) dx$$

$$=0-\int_0^{\frac{\pi}{4}}\tan^3 x\left(\tan^2 x-1\right)\left(\tan^2 x+1\right) dx$$

$$=\int_0^{\frac{\pi}{4}}\left(\tan^3 x-\tan^5 x\right)\sec^2 x dx=\left.\frac{\tan^4 x}{4}-\frac{\tan^6 x}{6}\right|_0^{\frac{\pi}{4}}$$

$$=\frac{1}{12}$$

Hence $7I_1+12I_2=1$

Q10.
(3) Put $\ln x = t \Rightarrow \frac{1}{x}dx = dt$ $\begin{array}{c|c} x & t \\ e^2 & 2 \\ e^4 & 4 \end{array}$

$I = \int_2^4 \frac{e^{(t^2+1)^{-1}}}{e^{(t^2+1)^{-1}} + e^{((6-t)^2+1)^{-1}}}dt \quad \ldots \text{(i)}$

$I = \int_2^4 \frac{e^{((6-t)^2+1)^{-1}}}{e^{((6-t)^2+1)^{-1}} + e^{(t^2+1)^{-1}}dt} \ldots \text{(ii)}$

$\left\{\text{Using} \int_a^0 f(x)dx = \int_a^0 f(a+b-x)dx\right\}$

Adding (i) and (ii) gives

$2l = \int dt \Rightarrow l = 1$

Q11.
(3) $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}dx}{(\sin x)^{\frac{3}{2}}x + (\cos x)^{\frac{3}{2}}} = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)dx}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}$

$\Rightarrow$ Adding $2l = \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}} + (\cos x)^{\frac{3}{2}}}{(\sin x)^{\frac{3}{2}} + (\cos x)^{\frac{3}{2}}}dx = \frac{\pi}{2}$

$I_0 = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}dx = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sin x \cos x}{(\sin x)^4 + (\cos x)^4}dx$

Adding, $2I_0 = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}(\sin x)\cos x}{\left(\sin^4 x + \cos^4 x\right)}dx$

$\Rightarrow I_0 = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{\tan x\left(\sec^2 x\right)dx}{1+\tan^4 x}$

put $\tan^2 x = t \Rightarrow 2\tan x \sec^2 x dx = dt$

$\Rightarrow I_0 = \frac{\pi}{4}\int_0^{\infty} \frac{\frac{dt}{2}}{(1+t^2)} = \frac{\pi}{8}\left(\tan^{-1} t\right)\Big|_0^{\infty} = \frac{\pi}{8}\left(\frac{\pi}{2}-0\right)$

$\Rightarrow I_0 = \frac{\pi^2}{16}$

Q12.
(4) $I = \int_0^{\frac{\pi}{2}} \frac{96x^2\cos^2 x}{1+e^x}dx$

$2I = 2\int_0^{\frac{\pi}{2}} 96x^2\cos^2 x dx$

$I = 96\int_0^{\frac{\pi}{2}} x^2\cos^2 x dx$

$= 48\int_0^{\frac{\pi}{2}} x^2(1+\cos 2x)dx$

$= 2\pi^2 + 48(0-0) - 48\int_0^{\frac{\pi}{2}} x\sin 2x dx$

$= 2\pi^2 - 12\pi + [0-0] = \pi\left(2\pi^2 - 12\right)$

$= \pi\left(\alpha\pi^2 + \beta\right)$

$\Rightarrow (\alpha+\beta)^2 = 100$

Q13. Given, $\int_0^1 f(\lambda x) d\lambda = a f(x)$

(112) Let $\lambda x = u$

$d\lambda = \frac{1}{x} du$

$\therefore$ From (1) $\frac{1}{x} \int_0^x f(u) du = a f(x)$

$\Rightarrow \int_0^x f(u) du = \text{axf}(x)$

Differentiate both sides

$f(x) = a\left(x f'(x) + f(x)\right)$

$\Rightarrow \quad f(x) = a x f'(x) + a f(x)$

$\Rightarrow \quad (1-a) f(x) = a x f'(x)$

$\Rightarrow \quad \frac{f'(x)}{f(x)} = \frac{(1-a)}{a} \cdot \frac{1}{x}$

Integrate both side w.r.t. (x)

$\Rightarrow \int \frac{f'(x)}{f(x)} dx = \frac{(1-a)}{a} \int \frac{1}{x} dx$

$\Rightarrow \ln f(x) = \left(\frac{1-a}{a}\right) \ln x + c$

Now at $x = 1 f(1) = 1$

$\Rightarrow c = 0$

Also given $f(16) = \frac{1}{8}$

$\therefore \quad \frac{1}{8} = (16)^{\frac{1-a}{a}}$

$\Rightarrow \quad 2^{-3} = 2^{\frac{4-4a}{a}}$

$\Rightarrow \quad -3 = \frac{4-4a}{a}$

$\Rightarrow -3a = 4 - 4a$

$\Rightarrow a = 4$

$\therefore \quad f(x) = x^{-3/4}$

$f(x) = \frac{-3}{4} x^{-\frac{7}{4}}$

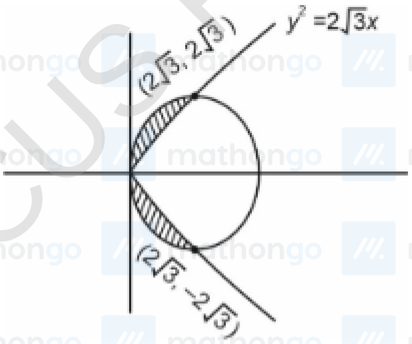
Put $x = \frac{1}{16}$

$f'\left(\frac{1}{16}\right) = \frac{-3}{4}\left(\frac{1}{16}\right)^{-7/4} = \frac{-3}{4} \cdot 2^{-4x\left(\frac{-7}{4}\right)} = -96$

$\therefore \quad 16 - f'\left(\frac{1}{16}\right) \Rightarrow 16 - (-96) = 112$

Q1.
(2)

Sol.



$$\text{Required area } = 2\int_0^{2\sqrt{3}}\left(\sqrt{4\sqrt{3}x-x^2}-\sqrt{2\sqrt{3}x}\right)dx$$

$$= 2\int_0^{2\sqrt{3}}\left(\sqrt{12-(x-2\sqrt{3})^2}-\sqrt{2\sqrt{3}x}\right)dx$$

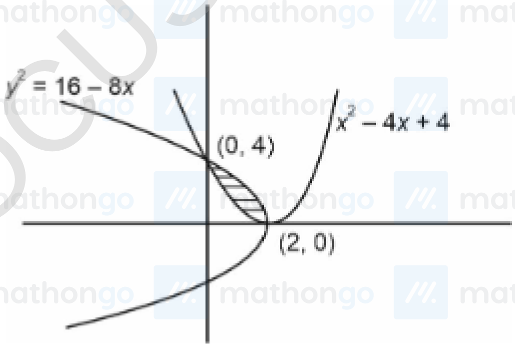
$$= 2\left[\frac{x-2\sqrt{3}}{2}\sqrt{12-(x-2\sqrt{3})^2}+\frac{12}{2}\sin^{-1}\left(\frac{x-2\sqrt{3}}{2\sqrt{3}}\right)\right.$$

$$\left.-\frac{\sqrt{2\sqrt{3}}x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{2\sqrt{3}}$$

$$= 2\{3\pi-8\}$$

$$= 6\pi-16 \text{ sq. units.}$$

Q2.
(1)

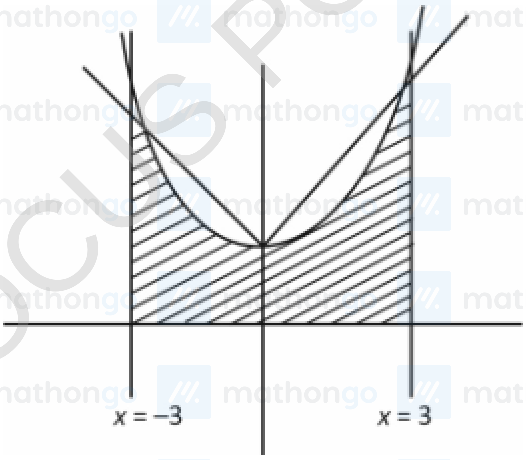


$$\text{Area } = \int_0^2\left(\sqrt{16-8x}-\left(x^2-4x+4\right)\right)dx$$

$$= \frac{-(16-8x)^{3/2}}{12}-\frac{x^3}{3}+2x^2+4x\Big]_0^2$$

$$= \frac{8}{3}$$

Q3.
(2)



$$\text{Area } = 2\left[\int_0^2 \left(x^2+1\right) dx + \frac{1}{2}[5+7] \times 1\right]$$
$$= \frac{64}{3}$$

Q4. $x\left(1+y^2\right)=1$

(2) $y^2=2x$

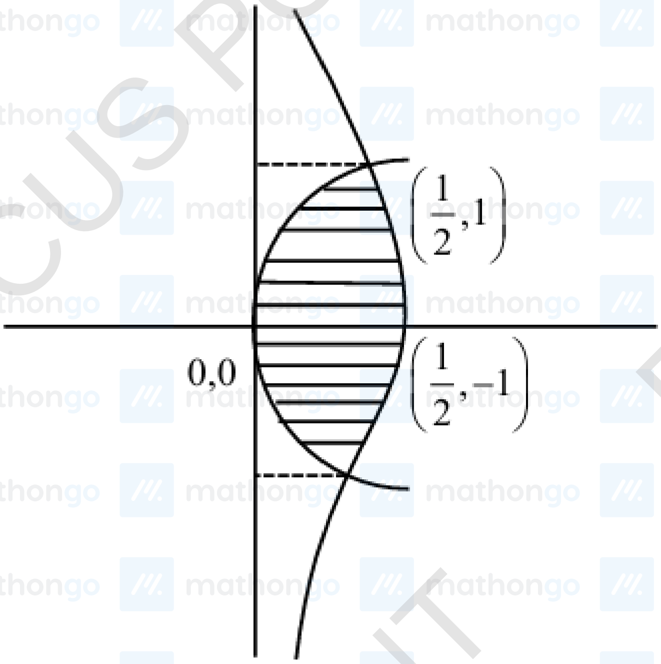
From equation (1) \& (2)

$x(1+2x)=1 \Rightarrow 2x^2+x-1=0$

$\Rightarrow x=\frac{1}{2}, x=-1$ (Reject)

$\Rightarrow y^2=2\left(\frac{1}{2}\right)$

$\Rightarrow y=\pm 1$



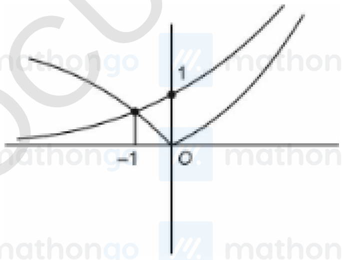
$$\text{Area bounded } = \int_{-1}^{1}\left(\frac{1}{1+y^2}-\frac{y^2}{2}\right)dy$$

$$=\left(\tan^{-1}y-\frac{y^3}{6}\right)\Bigg|_{-1}^{1}$$

$$=\frac{\pi}{2}-\frac{1}{3}$$

Q5.

(1)



$$e^x=1-e^x\Rightarrow 2e^x=1$$

$$\Rightarrow e^x=\frac{1}{2}$$

$$\Rightarrow x=\ln\frac{1}{2}$$

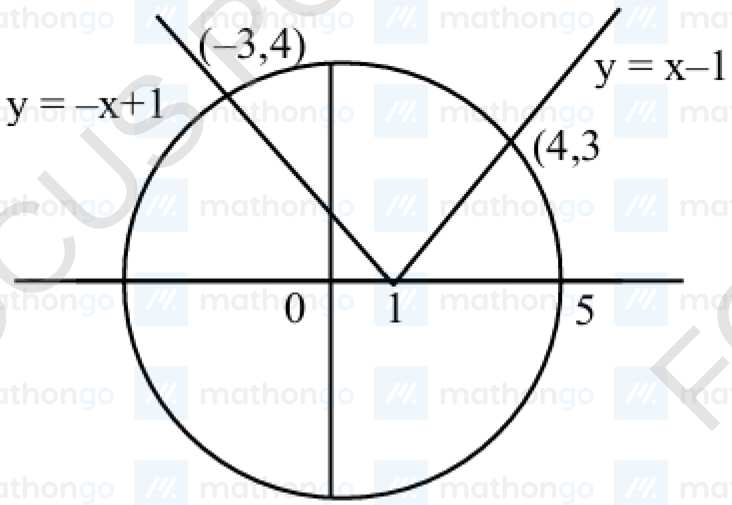
$$\int_{\ln(1/2)}^{0}\left[e^x-(1-e^x)\right]dx$$

$$=\int_{\ln 2}^{0}(2e^x-1)\,dx=2e^x-x|_{-\ln 2}^{0}$$

$$=2-(1+\ln 2)$$

$$=1-\log_e 2$$

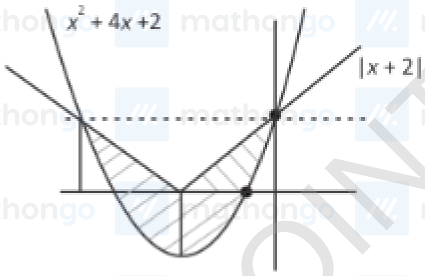
Q6.
(77)

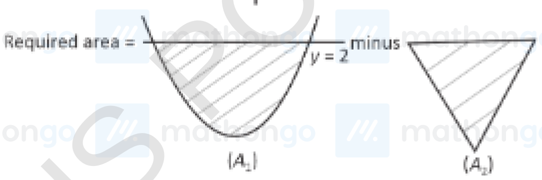


$x^2 + y^2 = 5$
$x^2 + (x-1)^2 = 25 \Rightarrow x = 4$
$x^2 + (-x+1)^2 = 5 \Rightarrow x = -3$
$A = 25\pi - \int_{-3}^{4} \sqrt{25 - x^2}dx + \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times 3 \times 3$
$A = 25\pi + \frac{25}{2} - \left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^{4}$
$A = 25\pi + \frac{25}{2} - \left[6 + \frac{25}{2}\sin^{-1}\frac{4}{5} + 6 + \frac{25}{2}\sin^{-1}\frac{3}{5}\right]$
$A = 25\pi + \frac{1}{2} - \frac{25}{2} \cdot \frac{\pi}{2}$
$A = \frac{75\pi}{4} + \frac{1}{2}$
$A = \frac{1}{4}(75\pi + 2)$
$b = 75, c = 2$
$b + c = 75 + 2 = 77$

Q7. $x^2 + 4x + 2 \leq y \leq |x + 2|$

(4)





$$A_1 = \int_{-4}^{0} \left[2 - \left(x^2 + 4x + 2\right)\right] dx - \frac{1}{2} \times 4 \times 2$$

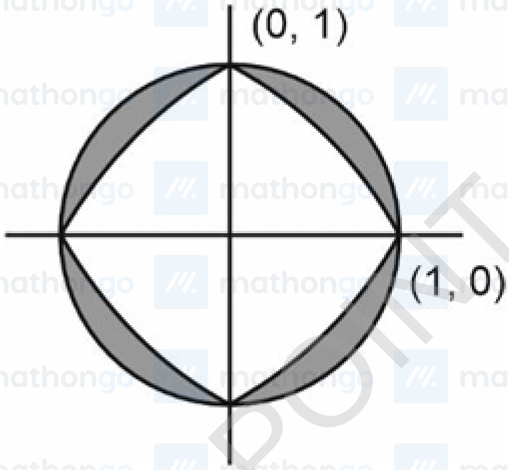
$$= \left(\frac{-x^3}{3} - 2x^2\right)_{-4}^{0} - 4$$

$$= 0 - \left(\frac{64}{3} - 32\right) - 4$$

$$= 32 - \frac{64}{3} - 4 = \frac{20}{3}$$

Q8.

(2)



$$\text{Required area } = \pi - 4\int_0^1 \left(1 - x^2\right) dx$$

$$= \pi - 4\left[x - \frac{x^3}{3}\right]_0^1$$

$$= \pi - 4 \times \frac{2}{3} = \pi - \frac{8}{3}$$

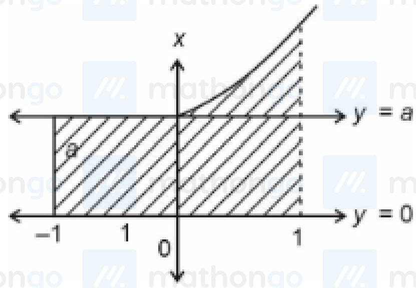
$$\therefore \alpha = \pi - \frac{8}{3}$$

$9\alpha = 9\pi - 24 \rightarrow \beta = 9, \gamma = -24$

$|\beta - \gamma| = |9 + 24| = 33$

Q9.

(3)



$y \in \left[0, a + e^{|x|} - e^{-x}\right]$

(i) If $x \geq 0 \Rightarrow y \in \left(0, a + e^x - \frac{1}{e^x}\right)$

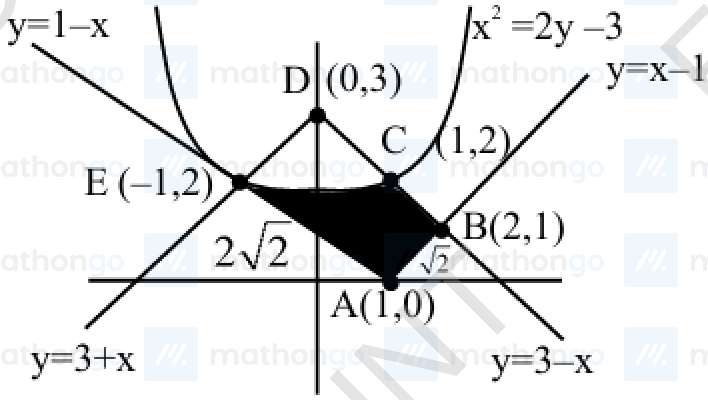
if $x < 0 \Rightarrow y \in \left(0, a + e^{-x} - e^{-x}\right)$

$\Rightarrow y \in (0, a)$

$$\text{Area } = (a) + \int_0^1 \left(a + e^x - e^{-x}\right) dx = \frac{e^2 + 8e + 1}{e}$$
$$= a + \left(ax + e^x + e^{-x}\right)\Big|_0^1 = e + 8 + \frac{1}{e}$$
$$= a + \left(a + e + \frac{1}{e} - 2\right) = e + \frac{1}{e} + 8$$
$$\Rightarrow 2a - 2 = 8 \Rightarrow a = 5$$

Q10.

(3)

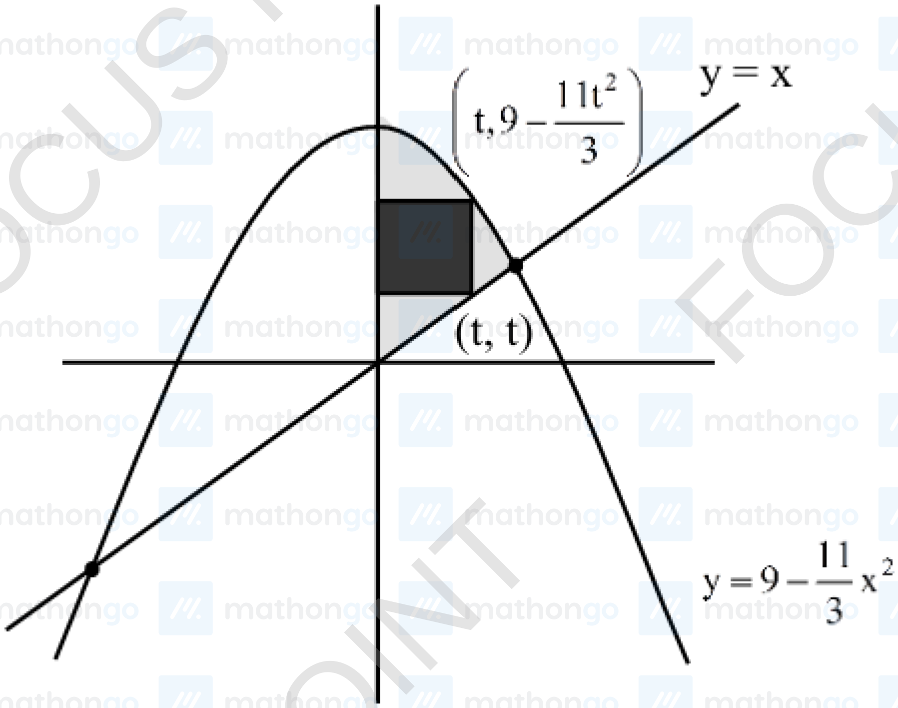


$A \Rightarrow$ Rectangle ABDE - Area of region EDC

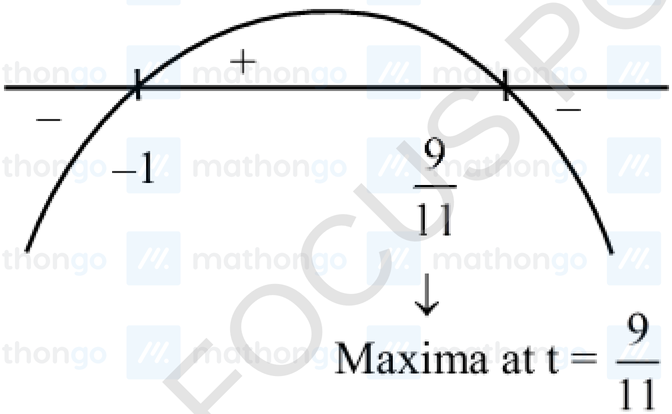
$$A \Rightarrow 4 - 2\int_0^1 (3 - x) - \left(\frac{x^2 + 3}{2}\right) dx$$
$$A \Rightarrow 4 - 2\left\{3x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{3}{2}x\right\}_0^1$$
$$A \Rightarrow 4 - 2\left\{3 - \frac{1}{2} - \frac{1}{6} - \frac{3}{2}\right\} = \frac{7}{3}$$

So 6 A = 14

Q11. t. $\left(9-\frac{11t^2}{3}-t\right)$
(4)



$\therefore \frac{dA}{dt} =$



$A = 9t - t^2 - \frac{11}{3}t^3$
$\frac{dA}{dt} = 9 - 2t - 11t^2$
$\Rightarrow 11t^2 + 2t - 9 = 0$
$11t^2 + 11t - 9t - 9 = 0$
$t = -1 \& t = \frac{9}{11}$

$\therefore$ largest area $= \frac{9}{11}\left(9 - \frac{11}{3}, \frac{81}{121} - \frac{9}{11}\right)$
$= \frac{9}{11} \cdot \frac{63}{11} = \frac{567}{121}$

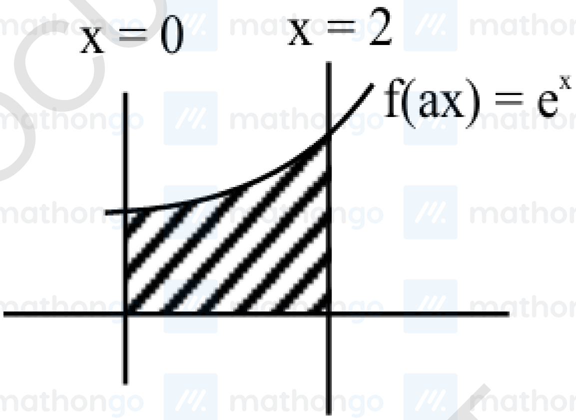
Q12. $f(x+y) = f(x) \cdot f(y)$
(1) $\Rightarrow f(x) = e^{\lambda x} f'(0) = 4a$
$\Rightarrow f'(x) = \lambda e^{\lambda x} \Rightarrow \lambda = 4a$
So, $f(x) = e^{4x}$

mathongo FOCUS POINT

$f''(x) - 3af'(x) - f(x) = 0$

$\Rightarrow \lambda^2 - 3a\lambda - 1 = 0$

$\Rightarrow 16a^2 - 12a^2 - 1 = 0 \Rightarrow 4a^2 = 1 \Rightarrow a = \frac{1}{2}$



$F(x) = e^{2x}$

$\text{Area } = \int_0^2 e^x dx = e^2 - 1$

Q1.
(1)

$\frac{dy}{dx}+2y\tan x=\sin x$

I.F. $=e^{2\int\tan x dx}=\sec^2 x$

$y\sec^2 x=\int\frac{\sin x}{\cos^2 x}dx$

$=\int\tan x\sec x dx$

$=\sec x+C$

$C=-2$

$y=\cos x-2\cos^2 x$

$y\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}-1$

$y'=-\sin x+4\cos x\sin x$

$y'\left(\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}+2$

$y'\left(\frac{\pi}{4}\right)+y\left(\frac{\pi}{4}\right)=1$

Q2.
(4)

$ydy=(xdy-ydx)\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y}=\left(\frac{xdy-ydx}{y^2}\right)\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y}=\sin\left(\frac{x}{y}\right)d\left(-\frac{x}{y}\right)$

$\rightarrow\ell ny=\cos\frac{x}{y}+C$

$x(1)=\frac{\pi}{2}\Rightarrow 0=\cos\frac{\pi}{2}+C\Rightarrow C=0$

lny $=\cos\frac{x}{y}$

but $y=2\Rightarrow\cos\frac{x}{2}=\ell n2$

$\cos x=2\cos^2\frac{x}{2}-1$

$=2(\ln 2)^2-1$

Q3.
(3)

$y^2dx+\left(x-\frac{1}{y}\right)dy=0$

$y^2dx=\left(\frac{1}{y}-x\right)dy$

$\Rightarrow y^2\frac{dx}{dy}=\frac{1}{y}-x$

$\Rightarrow\frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$

I.F. $=e^{\int\frac{1}{y^2}dy}=e^{\frac{-1}{y}}$

$\therefore$ Solution is

$xe^{\frac{-1}{y}}=\int e^{-\frac{1}{y}}\times\frac{1}{y^3}dy+C$

Let $\frac{-1}{y}=t$

$\Rightarrow \frac{1}{y^2}dy=dt$

$\Rightarrow xe^{-\frac{1}{y}}=-\int e^t t dt+C$

$\Rightarrow xe^{-\frac{1}{y}}=-e^t(t-1)+C$

$\Rightarrow xe^{-\frac{1}{y}}=-e^{\frac{-1}{y}}\left(\frac{-1}{y}-1\right)+C$

$x(1)=1$

$\Rightarrow e^{-1}=-e^{-1}(-2)+C$

$\Rightarrow C=-e^{-1}$

$\Rightarrow x=\frac{1}{y}+1-e^{-1+\frac{1}{y}}$

$x\left(\frac{1}{2}\right)=3-e$

Q4. $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{2e^{\tan^{-1}y}}{1+y^2}$

(4)

I.F. $=e^{\tan^{-1}y}$

$xe^{\tan^{-1}y}=\int\frac{2\left(e^{\tan^{-1}y}\right)^2dy}{1+y^2}$

Put $\tan^{-1}y=t, \frac{dy}{1+y^2}=dt$

$xe^{\tan^{-1}y}=\int 2e^{2t}dt$

$xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c$

$x=e^{\tan^{-1}y}+ce^{-\tan^{-1}y}$

$\because y=0, x=1$

$1=1+c\Rightarrow c=0$

$y=\frac{1}{\sqrt{3}}, x=e^{\pi/6}$

Q5. $\frac{dy}{dx}=\frac{2(3+y)\cdot e^{2x}}{7+e^{2x}}$
(3)
$\frac{dy}{dx}-\frac{2ye^{2x}}{7+e^{2x}}=\frac{6\cdot e^{2x}}{7+e^{2x}}$

I.F. $=e^{-\int\frac{2e^{2x}}{7+e^{2x}}dx}$

$\Rightarrow e^{-\ln\left(7+e^{2x}\right)}$

$=\frac{1}{7+e^{2x}}$

$y\cdot\frac{1}{7+e^{2x}}=\int\frac{6e^{2x}}{\left(7+e^{2x}\right)^2}dx$

$\frac{y}{7+e^{2x}}=\frac{-3}{7+e^{2x}+C}$

$\therefore y(0)=5$

$\Rightarrow\frac{5}{8}=\frac{-3}{8}+C$

$\Rightarrow C=1$

$\therefore y=-3+7+e^{2x}$

$y=e^{2x}+4$

$\therefore k=8$

Q6. $\left(1+x^2\right)\frac{dy}{dx}+xy=5x^1\sqrt{1+x^2}$
(2)
$\frac{dy}{dx}+\frac{xy}{1+x^2}=\frac{5x^2}{\sqrt{1+x^2}}$

$\therefore$ I.F. $=e^{\int\frac{x}{1+x^2}dx}=e^{\frac{\ln\left(1+x^2\right)}{2}}=\sqrt{1+x^2}$

$\therefore y\sqrt{1+x^2}=\int\frac{5x^2}{\sqrt{1+x^2}}\cdot\sqrt{1+x^2}dx$

$\therefore y\sqrt{1+x^2}=\int\frac{5x^2}{\sqrt{1+x^2}}\cdot\sqrt{1+x^2}dx$

$y\sqrt{1+x^2}=\frac{5x^3}{3}+C$

$\because y(0)=0\Rightarrow 0=0+C\Rightarrow C=0$

$\therefore y=\frac{5x^3}{3\sqrt{1+x^2}}$

$y(\sqrt{3})=\frac{15\sqrt{3}}{32}=\frac{5\sqrt{3}}{2}$

Q7. $2(x+2)^2f(x)-3(x+2)^2=10\int_0^x(t+2)f(t)dt$
(19) Differentiating both side

$$4(x+2)f(x)+2f(x)(x+2)^2-6(x+2)=10(x+2)f(x)$$

$$=(x+2)\frac{dy}{dx}-3y=3$$

$$\frac{1}{3}\int\frac{dy}{y+1}=\int\frac{dx}{x+2}$$

$$\ln|y+1=3\ln|x+2|+\ln c$$

$$y+1=(x+2)^3c$$

$$\because y(0)=\frac{3}{2}$$

$$\Rightarrow\frac{5}{16}=c$$

$$\therefore\quad y=\frac{5}{16}(x+2)^3-1$$

$$y(2)=\frac{5}{16}\times 64-1=19$$

Q8. $x^2f'(x)-2xf(x)=3$

(1) $\left(\frac{x^2f'(x)-2xf(x)}{(x^2)^2}\right)=\frac{3}{(x^2)^2}$

$$\Rightarrow\frac{d}{dx}\left(\frac{f(x)}{x^2}\right)=\frac{3}{x^4}$$

Integrating both sides

$$\frac{f(x)}{x^2}=-\frac{1}{x^3}+C$$

$$f(x)=-\frac{1}{x}+Cx^2$$

put $x=1$

$$4=-1+C\Rightarrow C=5$$

$$f(x)=-\frac{1}{x}+5x^2$$

Now $2\times f(2)=2\times\left[-\frac{1}{2}+5\times 2^2\right]$

$$=39$$

Q9.

(4) $\frac{dy}{dx}+\frac{\left(\sin^{-1}\frac{x}{2}\right)}{\sqrt{4-x^2}}y=\frac{\left(\sin^{-3}\frac{x}{2}\right)^3}{\sqrt{4-x^2}}$

$$ye^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}=\int\frac{\left(\sin^{-3}\frac{x}{2}\right)^3}{4-x^2}e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}dx$$

$$y=\left(\sin^{-1}\frac{x}{2}\right)^2-2+c\cdot e^{\frac{-\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}$$

$$y(2)=\frac{\pi^2}{4}-2\Rightarrow c=0$$

$$y(0)=-2$$

Q10. I.F. $\mathrm{e}^{-\frac{1}{2}\int\frac{2x}{1-x^2}dx}=\mathrm{e}^{-\frac{1}{2}\ln(1-x^2)}=\sqrt{1-x^2}$

(27) $y\times\sqrt{1-x^2}=\int\left(x^6+4x\right)dx=\frac{x^7}{7}+2x^2+c$

Given $y(0)=0 \Rightarrow c=0$

$y=\frac{\frac{x^7}{7}+2x^2}{\sqrt{1-x^2}}$

Now, $6\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\frac{x^7}{7}+2x^2}{\sqrt{1-x^2}}dx = 6\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{2x^2}{\sqrt{1-x^2}}dx$

$=24\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$

Put $x=\sin\theta$

$dx=\cos\theta d\theta$

$=24\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta$

$=24\int_0^{\frac{\pi}{6}} \left(\frac{1-\cos 2\theta}{2}\right)d\theta = 12\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{6}}$

$=12\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)$

$=2\pi-3\sqrt{3}$

$\alpha^2=(3\sqrt{3})^2=27$

Q11. (1) $\frac{dy}{dx}-\frac{3\sin x}{\cos x \ln\cos x}y=-\frac{\sin x}{\cos x(\ln\cos x)^2}$

I.F. $=e^{-\int \frac{3\tan x}{\ln\cos x}dx}$

Let $\ln\cos x=t$

$-\tan x dx=dt$

$e^{3\int \frac{dt}{t}}=e^{3\ln t}=t^3=(\ln\cos x)^3$

$\therefore$ Solution will be

$y(\ln\cos x)^3=-\int(\tan x)(\ln\cos x)dx$

$y(\ln\cos x)^3=\frac{(\ln\cos x)^2}{2}+c$

$\because \quad y\left(\frac{\pi}{4}\right)=-\frac{1}{\ln 2}$

$\Rightarrow \quad c=0$

$\therefore \quad y=\frac{1}{2(\ln\cos x)}$

$y\left(\frac{\pi}{6}\right)=\frac{1}{2}\times\frac{1}{\ln\left(\cos\frac{\pi}{6}\right)}$

$=\frac{1}{2}\left[\frac{1}{\ln\left(\frac{\sqrt{3}}{2}\right)}\right]$

$=\frac{1}{2}\times\frac{1}{\ln\sqrt{3}-\ln 2}$

$=\frac{1}{\ln 3-\ln 4}$

Q12. If $e^{\int \tan x dx} = e^{\ln(\sec x)} = \sec x$

(4) $\therefore y \cdot \sec x = \int \left\{ \frac{2+\sec x}{(1+2\sec x)^2} \right\} \sec x dx$

$= \int \frac{2\cos x + 1}{(\cos x + 2)^2} dx$ Let $\cos x = \frac{1-t^2}{1+t^2}$

$= \int \frac{2\left(\frac{1-t^2}{1+t^2}\right)+1}{\left(\frac{1-t^2}{1+t^2}+2\right)^2} 2dt$

$= \int \frac{2-2t^2+1+t^2}{(1-t^2+2+2t^2)^2} \times 2dt$

$= 2\int \frac{3-t^2}{(t^2+3)^2} dt$

Let $t + \frac{3}{t} = u$

$\left(1 - \frac{3}{t^2}\right) dt = du$

$= -2\int \frac{du}{u^2}$

$y \cdot (\sec x) = \frac{2}{u} + c$

$y \cdot \sec x = \frac{2}{t+\frac{3}{t}} + c \ldots (I)$

At $x = \frac{\pi}{3}, t = \tan\frac{x}{2} = \frac{1}{\sqrt{3}}$

$2 \cdot \frac{\sqrt{3}}{10} = \frac{2}{\frac{1}{\sqrt{3}}+3\sqrt{3}} + c$

$2. \frac{\sqrt{3}}{10} = \frac{2\sqrt{3}}{10} + c \Rightarrow C = 0$

At $x = \frac{\pi}{4}, t = \tan\frac{x}{2} = \sqrt{2}-1$

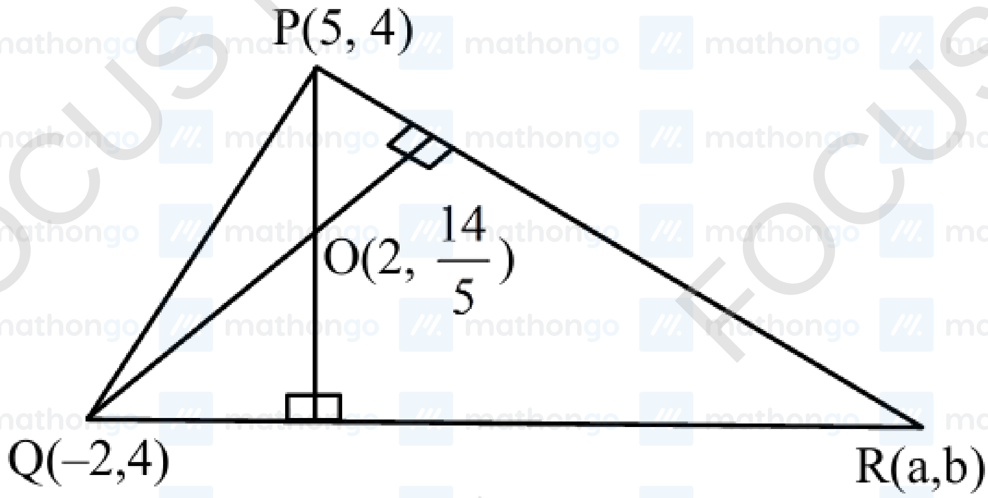
$\therefore y \cdot \sqrt{2} = \frac{2}{\sqrt{2}-1+\frac{3}{\sqrt{2}-1}}$

$y \cdot \sqrt{2} = \frac{2(\sqrt{2}-1)}{6-2\sqrt{2}}$

$y = \frac{\sqrt{2}(\sqrt{2}-1)}{2(3-\sqrt{2})} = \frac{1}{\sqrt{2}} \times \frac{2\sqrt{2}-1}{7}$

$= \frac{4-\sqrt{2}}{14}$

Q1.
(4)



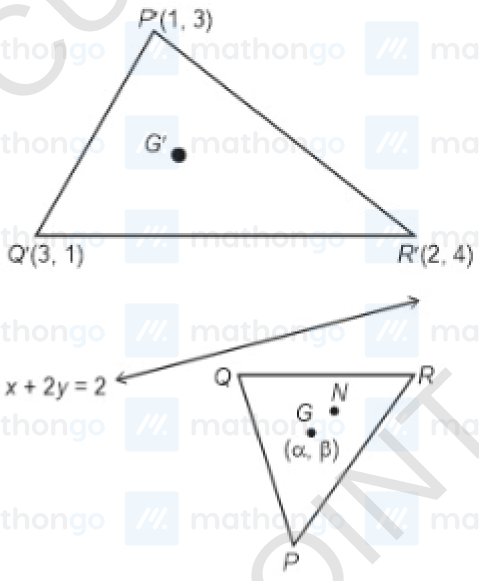
Equation of lines $QR = 5x + 2y + 2 = 0$
Equation of lines $PR = 10x - 3y - 38 = 0$
$\therefore$ Point $R(2, -6)$

Centroid $= \left(\frac{5-2+2}{3}, \frac{4+4-6}{3}\right)$

$= \left(\frac{5}{3}, \frac{2}{3}\right)$

$c + 2d = \frac{5}{3} + \frac{4}{3} = 3$

Q2. The centroid $G''(\alpha, \beta)$ of $\triangle PQR$ be image of centroid of given triangle $P'Q'R'$.
(4)



Centroid of $\Delta P'Q'R' = \left(\frac{1+3+2}{3}, \frac{3+1+4}{3}\right) = G'\left(2, \frac{8}{3}\right)$.
Image of $G\left(2, \frac{8}{3}\right)$, w.r.t. line $x + 2y = 2$ is (α, β)

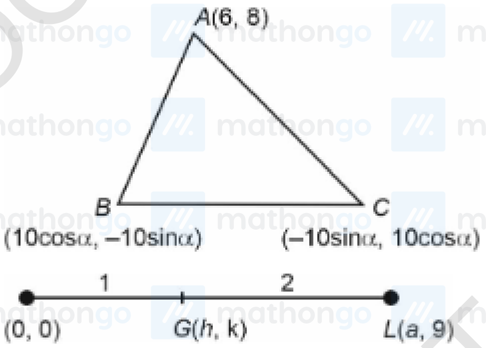
Then $\frac{\alpha-2}{1}=\frac{\beta-\frac{8}{3}}{2}=\frac{-2\left(2+\frac{16}{3}-2\right)}{1+4}$

$\therefore \frac{\alpha-2}{1}=\frac{\beta-\frac{8}{3}}{2}=-\frac{32}{15}$

$\therefore \quad \alpha=-\frac{2}{15}$ and $\beta=-\frac{8}{5}$

Then $15(\alpha-\beta)=15\left(-\frac{2}{15}+\frac{24}{15}\right)=22$

Q3.
(145)



$\frac{a+0}{3}=h \Rightarrow a=3h$

$\frac{9+0}{3}=k \Rightarrow k=3$

$\because (h,k)=\left(\frac{6+10\cos\alpha-10\sin\alpha}{3},\frac{8-10\sin\alpha-10\cos\alpha}{3}\right)$

$6+10\cos\alpha-10\sin\alpha=3h$

$10\cos\alpha-10\sin\alpha=3h-6$

$10(\cos\alpha-\sin\alpha)=1$

$\frac{8-10\sin\alpha+10\cos\alpha}{3}=k$

$\Rightarrow 100\sin 2\alpha=99$

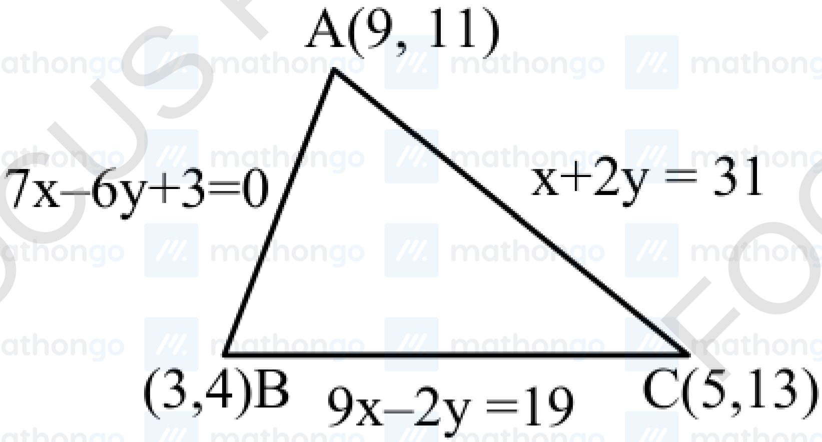
$h=\frac{7}{3}$

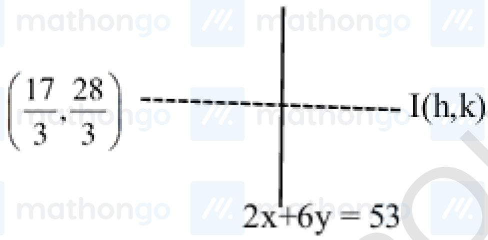
$\Rightarrow a=7$

Now, $5a-3h+6k+100\sin 2\alpha=35-7+18+99$

$=145$

Q4.
(2)





$\therefore$ centroid of $\triangle ABC = \left(\frac{9+3+5}{3}, \frac{11+4+13}{3}\right)$
$= \left(\frac{17}{3}, \frac{28}{3}\right)$

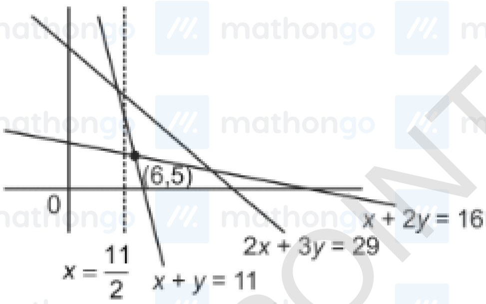
Let image of centroid with respect to line mirror is (h, k)

$\therefore \left(\frac{\mathrm{k}-\frac{28}{3}}{\mathrm{h}-\frac{17}{3}}\right)\left(-\frac{1}{2}\right) = -1$

$\&3\left(\frac{\mathrm{h}+\frac{17}{3}}{2}\right) + 6\cdot\left(\frac{\frac{\mathrm{k}+28}{3}}{2}\right) = 53$

Solving (1) \& (2) we get $\mathrm{h}=3, \mathrm{k}=4$
$\therefore \mathrm{h}^2+\mathrm{k}^2+\mathrm{hk}=37$

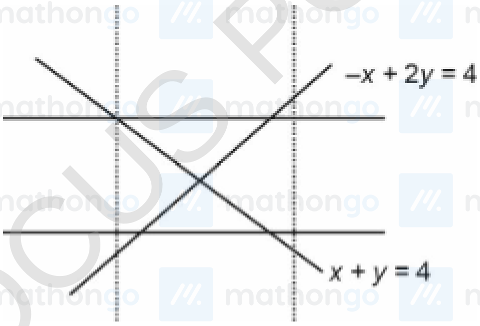
Q5.
(3)



Clearly, $x=\frac{11}{2}$ intersect $x+y-11=0$ at $\left(\frac{11}{2},\frac{11}{2}\right)$ and $2x+3y-29=0$ at $\left(\frac{11}{2},6\right) \Rightarrow \alpha = \left[\frac{11}{2},6\right]$

$\alpha_{\min}\cdot\alpha_{\max} = \frac{11}{2}\cdot 6 = 33$

Q6.
(3)



Slope of the third side = slope of the perpendicular bisector of given lines

$h : \frac{-x+2y-4}{\sqrt{5}} = \pm\frac{x+y-4}{\sqrt{2}}$

$h_1 : \sqrt{2}(-x+2y-4) = \sqrt{5}(x+y-4)$

$h_2 : \sqrt{2}(-x+2y-4) = -\sqrt{5}(x+y-4)$

$M_{L_1} : -\left[\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-2\sqrt{2}}\right]$

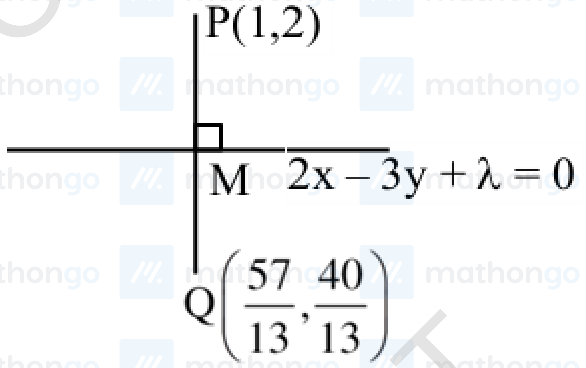
$M_{L_2} : -\left[\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+2\sqrt{2}}\right]$

$M_{L_1} + M_{L_2} = -\left[\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-2\sqrt{2}} + \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+2\sqrt{2}}\right]$

$= -\left[\frac{(\sqrt{5}+\sqrt{2})(\sqrt{5}+2\sqrt{2})+(\sqrt{5}-\sqrt{2})(\sqrt{5}-2\sqrt{2})}{-3}\right]$

$= 6$

Q7.
(3)



$\because$ PM = QM

So, M $\left(\frac{\frac{57}{13}+1}{2}, \frac{\frac{-40}{13}+2}{2}\right)$

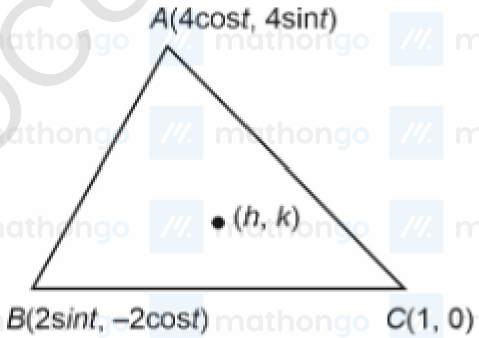
$= \left(\frac{35}{13}, \frac{-7}{13}\right)$

$\because$ M lies on the time

$$2x-3y+\lambda=0$$
$$2\left(\frac{35}{13}\right)-3\left(\frac{-7}{13}\right)+\lambda=0$$
$$\lambda=-\frac{70}{13}+\frac{21}{13}$$
$$=\frac{-91}{13}=-7$$
$$\begin{vmatrix}3 & -4 & -\alpha\\ 8 & -11 & -33\\ 2 & 3 & \lambda\end{vmatrix}=0$$
$$\Rightarrow 3(-11\lambda-99)+4(8\lambda+66)-\alpha(-24+22)=0$$
$$\Rightarrow 33\lambda-297+32\lambda+264+24\alpha-22\alpha=0$$
$$\Rightarrow -\lambda+2\alpha-33=0$$
$$\therefore \lambda=-7$$
$$-(-7)+2\alpha-33=0$$
$$2\alpha=26$$
$$\alpha=13$$
$$\therefore |\alpha\lambda|=|13\times(-7)|$$
$$=91$$

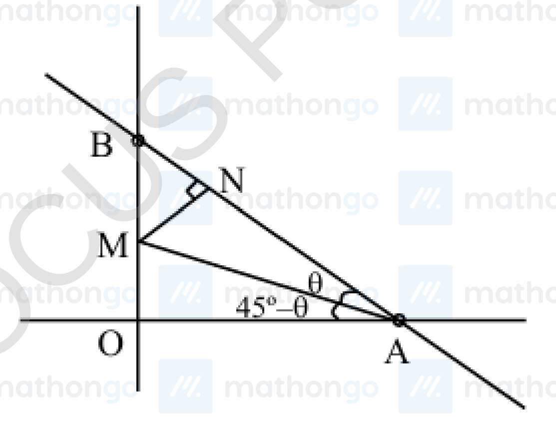
Q8.
(4)

$$\left.\begin{array}{l} {}^nC_{r-1}=28\\ {}^nC_r=56\\ {}^nC_{r+1}=70\\ \frac{{}^nC_{r-1}}{{}^nC_r}=\frac{28}{56}\Rightarrow\frac{r}{n-r+1}=\frac{1}{2}\\ \frac{{}^nC_r}{{}^nC_{r+1}}=\frac{56}{70}\Rightarrow\frac{r+1}{n-r}=\frac{70}{56}\end{array}\right\} n=8=3$$



$$h=\frac{4\cos t+2\sin t+1}{3}\quad k=\frac{4\sin t-2\cos t}{3}$$
$$3h-1=4\cos t+2\sin t$$
$$3k-1=4\sin t-2\cos t$$
$$(1)^2+(2)^2$$
$$(3h-1)^2+(3k)^2=20$$
Locus of centroid: $(3x-1)^2+(3y)^2=20$
$$\Rightarrow \alpha=20$$

Q9.
(1)



Area of $\triangle AOB = \frac{1}{2}$

Area of $\triangle AMN = \frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$

Equation of AB is $x + y = 1$

$OA = 1, AM = \sec(45^\circ - \theta)$

$AN = \sec(45^\circ - \theta)\cos\theta$

$MN = \sec(45^\circ - \theta)\sin\theta$

$Ar(\triangle AMN) = \frac{1}{2} \times \sec^2(45^\circ - \theta)\sin\theta \cdot \cos\theta = \frac{2}{9}$

$\Rightarrow \tan\theta = 2, \frac{1}{2}$

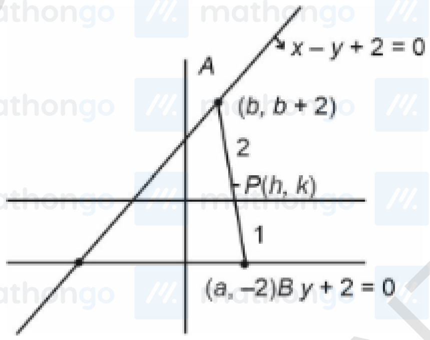
$\tan\theta = 2$ is rejected

$\frac{AN}{NB} = \frac{\lambda}{1} = \cot\theta = 2$

Q10.
(3)

$AB = 8$

$AB^2 = 64$



$\Rightarrow (a-b)^2 + (b+4)^2 = 64$ ...(1)

Now P divides AB in the ratio 2 : 1 internally

$\Rightarrow h = \frac{2a+b}{3}$ and $k = \frac{-4+b+2}{3}$

$\Rightarrow 2a + b = 3h \ldots (2)\ k = \frac{b-2}{3}$

From equation (2) and (3) $\quad \Rightarrow b = 3k + 2$

$\Rightarrow \quad 2a = 3h - 3k - 2$

$\Rightarrow \quad a = \dfrac{3h - 3k - 2}{2}$

Now by putting value of a and b in equation

$\Rightarrow \left(\dfrac{3h-3k-2}{2} - (3k+2)\right)^2 + (3k+2+4)^2 = 64$

$\Rightarrow \left(\dfrac{3h-3k-2-6k-4}{2}\right)^2 + (3k+6)^2 = 64$

$\Rightarrow (3h-9k-6)^2 + 4(3k+6)^2 = 4 \times 64$

$\Rightarrow 9(h-3k-2)^2 + 36(k+2)^2 = 256$

$\Rightarrow 9\left(h^2 + 9k^2 + 4 - 6hk - 4h + 12k\right)$

$+ 36\left(k^2 + 4 + 4k\right) = 256$

$\Rightarrow 9\left(h^2 + 13k^2 + 20 - 6hk - 4h + 28k\right) = 256$

Replacing h by x and k by y

$\Rightarrow 9\left(x^2 + 13y^2 - 6xy - 4x + 28y\right) + 180 - 256 = 0$

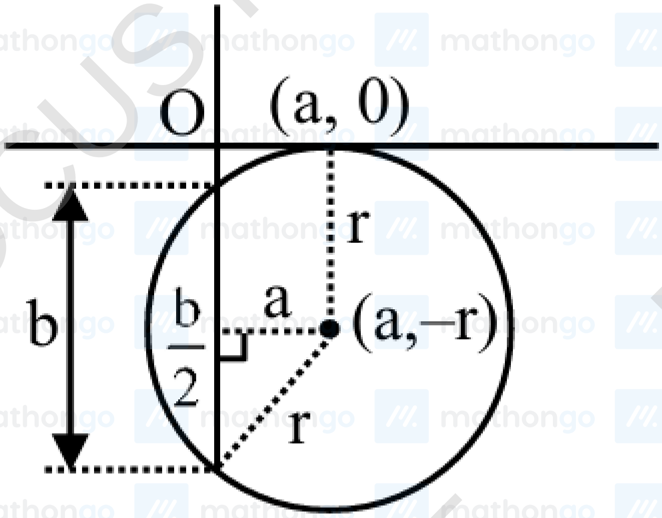
$\Rightarrow 9\left(x^2 + 13y^2 - 6xy - 4x + 28y\right) - 76 = 0$

By comparing $\alpha = 13, \beta = -6, \gamma = -4$

$\alpha - \beta - \gamma = 13 + 6 + 4 = 23$

FOCUS POINT mathongo

Q1.
(4)



By pytogorus $r^2 = a^2 + \frac{b^2}{4} = P^2$

$r = \sqrt{\frac{4a^2+b^2}{4}}$

Equation of circle is $(x-\alpha)^2 + (y-\beta)^2 = r^2$

$x^2 + y^2 - 2ax - 2py + \alpha^2 + p^2 - r^2 = 0$

comparision $x^2 + y^2 - \alpha x + \beta y + r = 0$

$-\alpha = -2a, \beta = -2p, r = a^2$

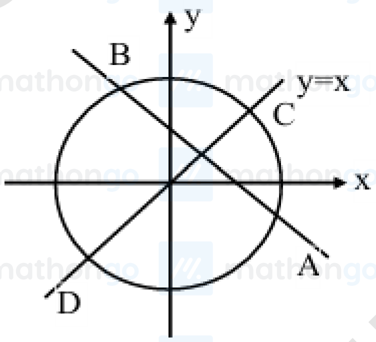
$\Rightarrow 2a = \alpha, 4a^2 + b^2 = 4p^2$

$\alpha^2 + b^2 = 4p^2$

$\alpha^2 + b^2 = \beta^2$

So, $\left(2a, b^2\right) = \left(\alpha, \beta^2 - 4r\right)$

Q2.
(3)



By solving $x = y$ with circle We get

$C(\sqrt{2}, \sqrt{2})$

$D(-\sqrt{2}, -\sqrt{2})$

By solving $x + y = 1$ with circle $x^2 + y^2 = 4$

we set

$A\left(\frac{1+\sqrt{7}}{2}, \frac{1-\sqrt{7}}{2}\right)$

$\& B\left(\frac{1-\sqrt{7}}{2}, \frac{1+\sqrt{7}}{2}\right)$

$\therefore$ Area of Quadrilateral ACBD

$= 2 \times$ Area of $\triangle BCD$

$= 2 \times \frac{1}{2}\begin{vmatrix} \sqrt{2} & \sqrt{2} & 1 \\ \frac{1-\sqrt{7}}{2} & \frac{1+\sqrt{7}}{2} & 1 \\ -\sqrt{2} & -\sqrt{2} & 1 \end{vmatrix}$

$= 2\sqrt{14}$

Q3. Let the centre be

(3) $\left(-2a, \frac{6a-2}{2}\right) \equiv (-2a, 3a-1)$

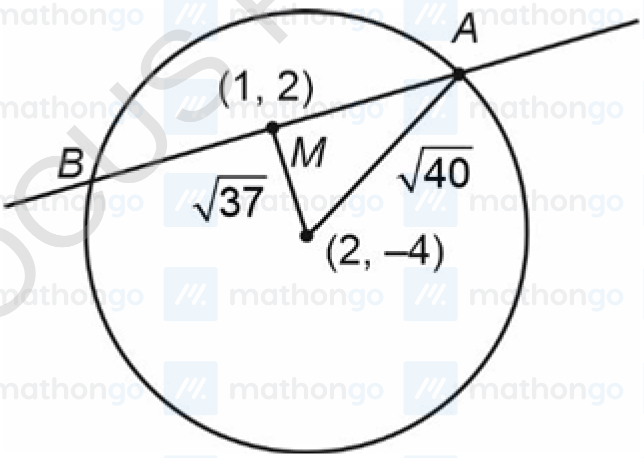
Centre is equal distance from $(4,2)$ and $(0,2)$

$\Rightarrow \sqrt{(4+2a)^2+(3a-3)^2} = \sqrt{(-2a-0)^2+(3a-3)^2}$

$\Rightarrow (2a+4)^2 + 9(a-1)^2 = 4a^2 + 9(a-1)^2$

$\Rightarrow 4a^2 + 16 + 16a = 4a^2 \Rightarrow a = -1$

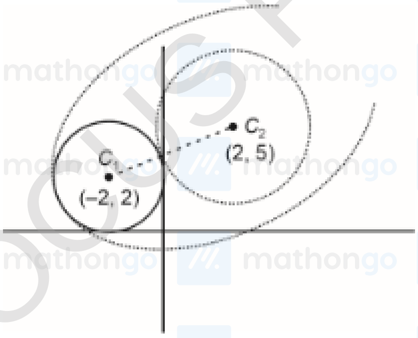
$\Rightarrow$ centre $\equiv (2,-4) \Rightarrow$ Radius $= \sqrt{40}$



$\Rightarrow AM^2 = (\sqrt{40})^2 - (\sqrt{37})^2$

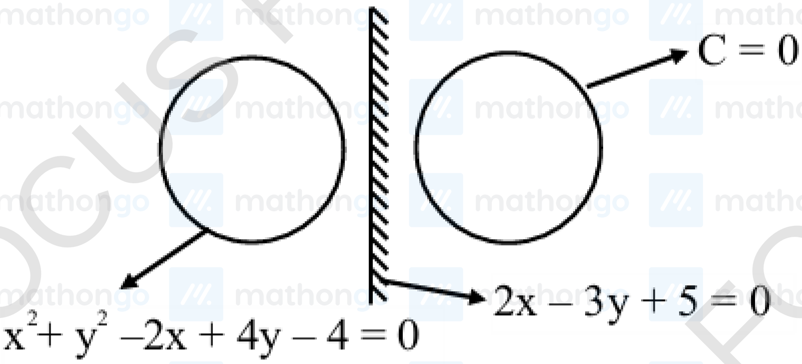
$\Rightarrow 2AM = AB = 2\sqrt{3}$

Q4.
(2)



$C_1C_2 = \sqrt{(2+2)^2 + (5-2)^2}$
$= \sqrt{16+9}$
$= 5$
$r + 2 > 5$
$r > 3$
$r < 5 + 2$
$r < 7$
$\therefore \alpha = 3, \beta = 7$
$3\beta - 2\alpha = 3(7) - 2(3) = 21 - 6 = 15$

Q5.
(2)



Centre $(1, -2), r = 3$
Reflection of $(1, -2)$ about $2x - 3y + 5 = 0$
$\frac{x-1}{2} = \frac{y+2}{-3} = \frac{-2(2+6+5)}{13} = -2$
$x = -3, y = 4$
Equation of circle ' C '
$C : (x+3)^2 + (y-4)^2 = 9$

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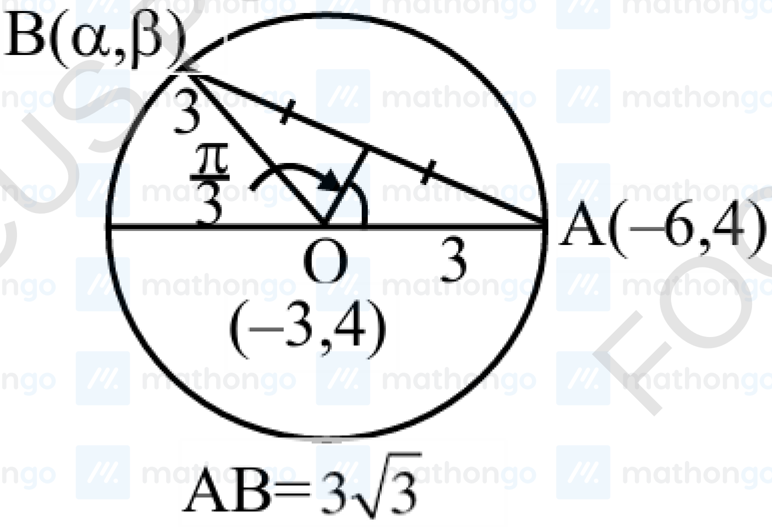

A.T.Q.

$\ell(\text{arcAB}) = \frac{1}{6} \times 2\pi r$

$r\theta = \frac{1}{6} \times 2\pi r$

$\theta = \frac{\pi}{3}$

$(\alpha+6)^2 + (\beta-4)^2 = 27$

$(\alpha+3)^2 \pm (\beta-4)^2 = 9$

$(\alpha+6)^2 - (\alpha+3)^2 = 18$

$\Rightarrow 6\alpha = -9$

$\Rightarrow \alpha = \frac{-3}{2}, \beta = \left(4 - \frac{3\sqrt{3}}{2}\right)$

$\therefore \beta - \sqrt{3}\alpha$

$\left(4 - \frac{3\sqrt{3}}{2}\right) + \frac{3\sqrt{3}}{2}$

$= 4$

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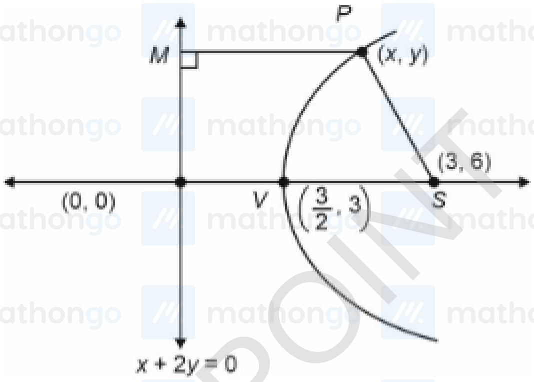
Q1. $y^2 = 4(x+4)$
(15) Equation of circle
$(x+3)^2 + y^2 = 25$
Passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$
$\left(\frac{4}{3\lambda+1}, \frac{12}{3\lambda+1}\right)$, we get
$\lambda = -\frac{7}{6}, 1$
$12\lambda_1 + 29\lambda_2$
$-14 + 29 = 15$

Q2.
(2)


$\therefore$ Equation of parabola is $PS = (1)PM$
$\Rightarrow PS^2 = PM^2$
$(x-3)^2 + (y-6)^2 = \left(\frac{x+2y}{\sqrt{5}}\right)^2$
$5x^2 - 30x + 5y^2 - 60y + 225 = x^2 + 4y^2 + 4xy$
$4x^2 + y^2 - 4xy - 30x - 60y + 225 = 0$
We get : $\alpha = 4, \beta = 1, \gamma = 4$
$\therefore \alpha + \beta + \gamma = 9$

Q3. The parabolas are
(3) $(x-4)^2 + (y-3)^2 = x^2$ ...(i)
and $(x-4)^2 + (y-3)^2 = y^2$...
If point of intersection are $A(x_1, y_1)$ and $B(x_2, y_2)$ By solving (i) and (ii), we get
$x_1 + x_2 = 14$ and $x_1 x_2 = 25$
$(AB)^2 = 2\left((x_1+x_2)^2 - 4x_1x_2\right) = 192$

Q4. $\mathrm{y} = \mathrm{x}^2 + \mathrm{px} - 3$
(3) Let $\mathrm{P}(\alpha, 0), \mathrm{Q}(\beta, 0), \mathrm{R}(0, -3)$
Circle with centre $(-1, -1)$ is $(x+1)^2 + (y+1)^2 = r^2$
Passes through $(0, -3)$
$1^2 + (-2)^2 = r^2]$
$\mathrm{r}^2 = 5$

$(x+1)^2+(y+1)^2=5$

Put $y=0$

$(x+1)^2=5-1$

$(x+1)^2=4$

$x+1=\pm 2$

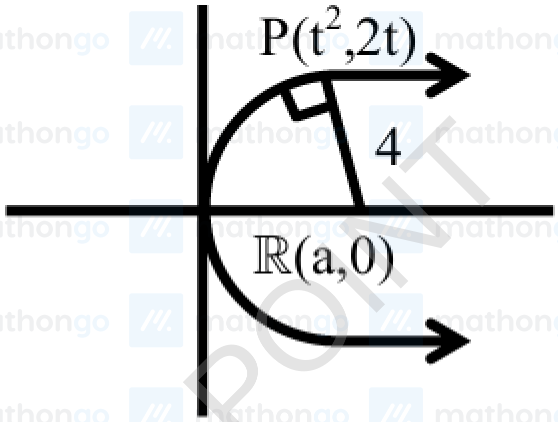
$x=1$ or $x=-3$

$\therefore P(1,0)$ and $Q(-3,0)$

Area of $\triangle PQR=\frac{1}{2}\begin{vmatrix}1 & 0 & 1\\ -3 & 0 & 1\\ 0 & -3 & 1\end{vmatrix}=6$

Q5.

(2)



Normal at P

$y+tx=2t+t^3$

$\uparrow$

$(a,0)$

$at=2t+t^3$

$a=2+t^2$

$\mathbb{R}\left(2+t^2,0\right)$

$PR=4\Rightarrow 4+4t^2=16$

$4t^2=12\Rightarrow t^2=3$

$a=5\mathbb{R}(5,0)$

Focus $(1,0)$

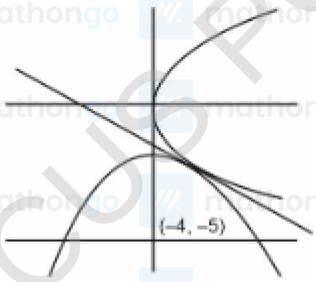
$(1,0)\&(5,0)$ will be tha end pts. of diameter

$\Rightarrow Eg^n$ of circle is

$(x-1)(x-5)+y^2=0$

$x^2+y^2-6x+5=0$

Q6.
(14)



To find image of $P\left(t^2, 2t\right)$

$$\frac{x-t^2}{1}=\frac{y-2t}{1}=\frac{-2\left(t^2+2t+4\right)}{1^2+1^2}=-(t+1)^2-3$$

$x=t^2-(t+1)^2-3=-2t-4$

$y=2t-(t+1)^2-3=-t^2-4$

$t=\frac{-x-4}{2}$

$\Rightarrow y+4=-\left(\frac{-x-4}{2}\right)^2$

$\Rightarrow (y+4)=-\frac{(x+4)^2}{4}$

$\Rightarrow x^2=-4y$

$\Rightarrow$ Focus $(-4,-5)$

Also, $y=-5$ intersect

$\therefore (-4)(-1)=(x+4)^2$

$4=(x+4)^2$

$x+4=\pm 2$

$x=-2,-6$

$\Rightarrow d=4$

$$a=\frac{1}{2}\left|\begin{array}{ccc}1 & 0 & 1\\ -2 & -5 & 1\\ -6 & -5 & 1\end{array}\right|$$

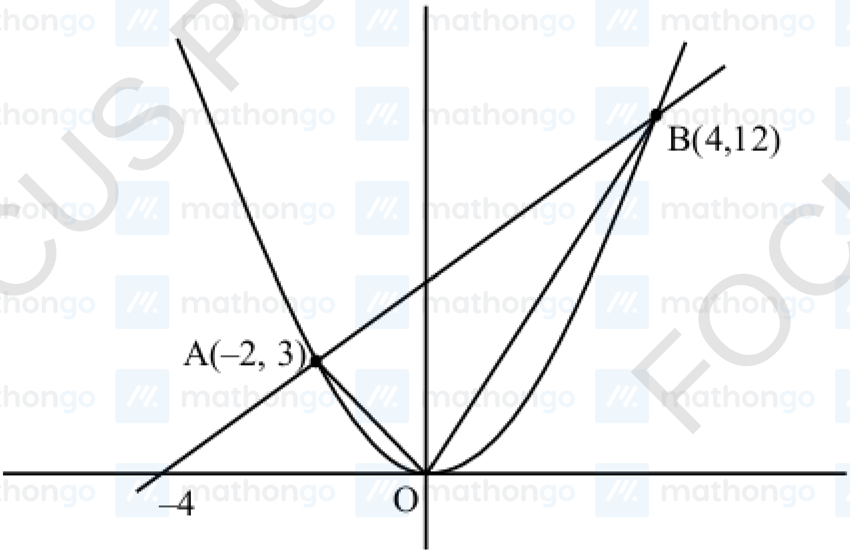
$=\frac{1}{2}|[1(-5+5)+1(10-30)]|$

$=\frac{1}{2}(20)$

$a=10$

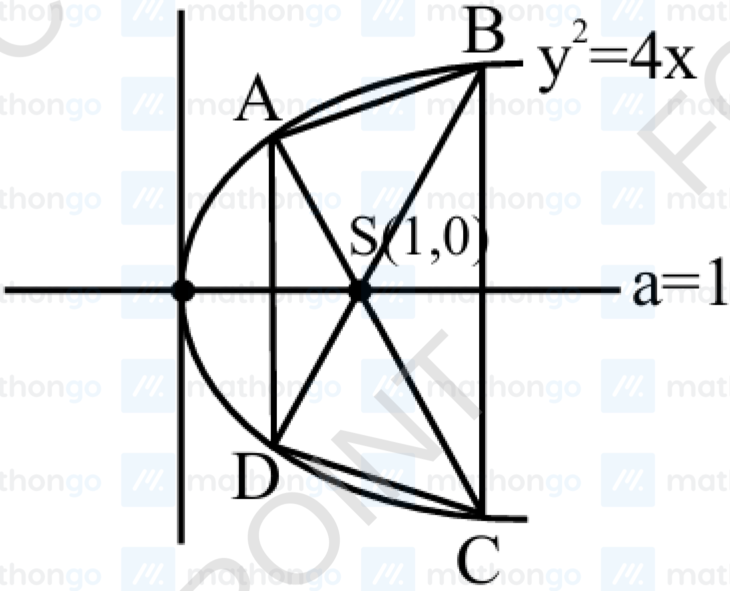
$\therefore \quad a+d=14$

Q7.
(2)



$3x-2y+12=0$
$4y=3x^2$
$\therefore 2(3x+12)=3x^2$
$\Rightarrow x^2-2x-8=0$
$\Rightarrow x=-2,4$
$\mathrm{m_{OA}}=-3/2,\ \mathrm{m_{OB}}=3$
$\tan\theta=\left(\frac{\frac{-3}{2}-3}{1-\frac{9}{2}}\right)=\frac{9}{7}$
$\theta=\tan^{-1}\left(\frac{9}{7}\right)$ (angle will be acute)

Q8.
(1)



FOCUS POINT
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$A\left(at_1^2, 2at\right) \& C\left(\frac{a}{t_1^2}, -\frac{2a}{t_1}\right)$

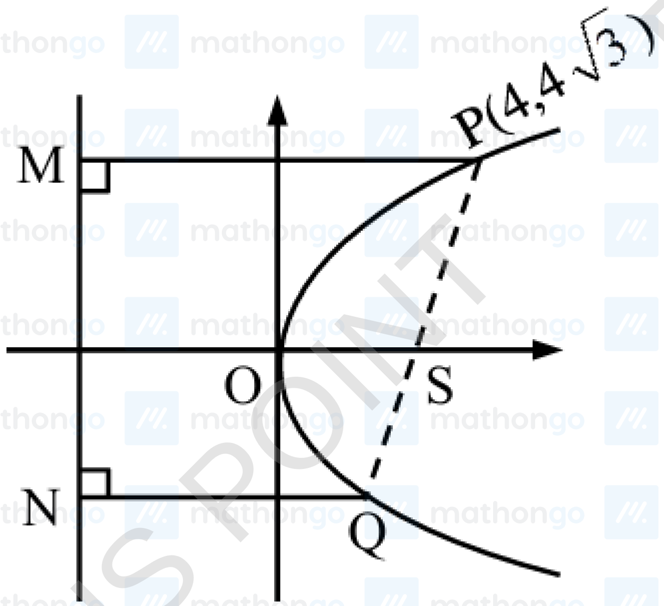
$\text{Length } AC = a\left(t_1 + \frac{1}{t_1}\right)^2 = \frac{25}{4}, t_1 + \frac{1}{t_1} = \pm\frac{5}{2}$

$\Rightarrow t_1 = 2 \text{ or } \frac{1}{2}, A\left(\frac{1}{2}, 1\right), D\left(\frac{1}{4}, -1\right), B(4,4), C(4,-4)$

So, area of trapezium $= \frac{1}{2}(8+2)\left(4 - \frac{1}{4}\right) = \frac{75}{4}$

Q9.

(4)



$(4, 4\sqrt{3})$ lies on $y^2 = 4ax$

$\Rightarrow 48 = 4a \cdot 4$

$4a = 12$

$\Rightarrow y^2 = 12x$ is equation of parabola

Now, parameter of P is $t_1 = \frac{2}{\sqrt{3}} \Rightarrow$ Parameters of Q is $t_2 = -\frac{\sqrt{3}}{2} \Rightarrow Q\left(\frac{9}{4}, -3\sqrt{3}\right)$

Area of trapezium PQNM

$= \frac{1}{2}MN \cdot (PM + QN)$

$= \frac{1}{2}MN \cdot (PS + QS)$

$= \frac{1}{2}MN \cdot PQ$

$= \frac{1}{2}7\sqrt{3} \cdot \frac{49}{4} = (343)\frac{\sqrt{3}}{8} = 3$

Q10. $y^2 = 12x \quad a = 3 \quad SP \times SQ = \frac{147}{4}$

(1328) Let $P\left(3t^2, 6t\right)$ and $t_1 t_2 = -1$

(ends of focal chord)

So, $Q\left(\frac{3}{t^2}, \frac{-6}{t}\right)$

$S(3,0)$

$SP \times SQ = PM_1 \times QM_2$

(dist. from directrix)

$= (3+3t^2)\left(3+\frac{3}{t^2}\right) = \frac{147}{4}$

$\Rightarrow \frac{(1+t^2)^2}{t^2} = \frac{49}{12}$

$t^2 = \frac{3}{4}, \frac{4}{3}$

$t = \pm\frac{\sqrt{3}}{2}, \pm\frac{2}{\sqrt{3}}$

considering $t = \frac{-\sqrt{3}}{2}$

$P\left(\frac{9}{4}, -3\sqrt{3}\right)$ and $Q(4, 4\sqrt{3})$

Hence, diametric circle:

$(x-4)\left(x-\frac{9}{4}\right) + (y+3\sqrt{3})(y-4\sqrt{3}) = 0$

$\Rightarrow x^2 + y^2 - \frac{25}{4}x - \sqrt{3}y - 27 = 0$

$\Rightarrow \alpha = 400, \beta = 1728$

$\beta - \alpha = 1328$

Q1.
(2)

Product of focal distances $= (a + ex_1)(a - ex_1)$

$= a^2 - e^2x_1^2 = a^2 - e^2(3)$

$= a^2 - 3e^2 = \frac{7}{4} \Rightarrow a^2 = \frac{7}{4} + 3e^2$

$\Rightarrow 4a^2 = 7 + 12e^2$

$\& \left(\sqrt{3}, \frac{1}{2}\right)$ lines on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore \frac{3}{a^2} + \frac{1}{4b^2} = 1$

$\frac{3}{\mathrm{a}^2} + \frac{1}{4(\mathrm{a}^2)(1 - \mathrm{e}^2)} = 1$

$12(1 - \mathrm{e}^2) + 1 = 4\mathrm{a}^2(1 - \mathrm{e}^2)$

$13 - 12\mathrm{e}^2 = (7 + 12\mathrm{e}^2)(1 - \mathrm{e}^2)$

$\Rightarrow 13 - 12\mathrm{e}^2 = 7 - 7\mathrm{e}^2 + 12\mathrm{e}^2 - 12\mathrm{e}^4$

$\Rightarrow 12\mathrm{e}^4 - 17\mathrm{e}^2 + 6 = 0$

$\therefore \mathrm{e}^2 = \frac{17 \pm \sqrt{289 - 288}}{24} = \frac{17 \pm 1}{24} = \frac{3}{4} \& \frac{2}{3}$

$\therefore \mathrm{e} = \frac{\sqrt{3}}{2} \& \sqrt{\frac{2}{3}}$

$\therefore$ difference $== \frac{\sqrt{3}}{2} - \sqrt{\frac{2}{3}} = \frac{3 - 2\sqrt{2}}{2\sqrt{3}}$

Q2.
(4)

$2ae = 4$

$\Rightarrow \quad a = 2\sqrt{3}$

$\Rightarrow \quad 1 - \frac{b^2}{12} = \frac{1}{3} \Rightarrow b^2 = 8$

$\frac{2b^2}{a} \times \frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow$

$\frac{2 \times 8}{2\sqrt{3}} \times \frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow$

$\frac{A^2}{B} = 2 \Rightarrow A^2 = 2B$

$1 - \frac{A^2}{B} = \frac{1}{3}$

$\Rightarrow$

$B = 3 \Rightarrow A^2 = 6$

$E_1 : \frac{x^2}{12} + \frac{y^2}{8} = 1$ ....(i)

$E_1 : \frac{x^2}{6} + \frac{y^2}{9} = 1$...(ii)

On solving (i) \& (ii)

$(x, y) = \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right),$
$\left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right)$

Four points are vertices of rectangle area $= \frac{24\sqrt{6}}{5}$

Q3. $T = S_1$
(3) $\frac{x \cdot 1}{4} + \frac{y}{4} = \frac{1}{4} + \frac{1}{8}$

$\Rightarrow 2x + 2y = 3$

$\frac{x^2}{4} + \frac{\left(\frac{3-2x}{2}\right)^2}{2} = 1$

$\Rightarrow x = \frac{12 \pm \sqrt{120}}{12} \Rightarrow y = \frac{1}{2} \mp \frac{\sqrt{120}}{12}$

So length of chord

$= \frac{2\sqrt{15}}{3}$

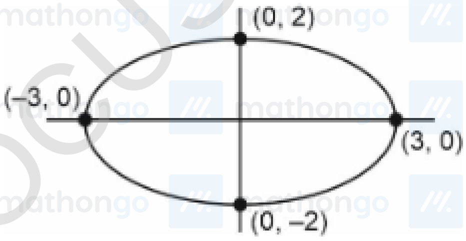
Q4. Equation of chord with given middle point
(1) $T = S_1$

$\Rightarrow \frac{3x}{25} + \frac{\mathrm{y}}{16} - 1 = \frac{9}{25} + \frac{1}{16} - 1$

$48\mathrm{x} + 25\mathrm{y} = 144 + 25$

$48\mathrm{x} + 25\mathrm{y} = 169$ Ans.

Q5.
(2)



$E : \frac{x^2}{9} + \frac{y^2}{4} = 1$

$T = S_1$

$\Rightarrow \frac{\sqrt{2}x}{9} + \frac{1}{4}\left(\frac{4}{3}y\right) - 1 = \frac{2}{9} + \frac{16}{9(4)} - 1$

$\frac{\sqrt{2}x}{9} + \frac{y}{3} = \frac{2}{9} + \frac{4}{9}$

$\frac{\sqrt{2}x}{9} + \frac{y}{3} = \frac{2}{3} \quad \Rightarrow \sqrt{2x} + 3y = 6$

Now point of intersection of chord and ellipse is

$$\frac{(6-3y)^2}{18}+\frac{y^2}{4}=1$$
$$\frac{(2-y)^2}{2}+\frac{y^2}{4}=1$$
$$2\left(4+y^2-4y\right)+y^2=4$$
$$\Rightarrow 3y^2-8y+4=0$$
$$\Rightarrow y=2,\frac{2}{3}$$

So, points are $(0,2)$ are $\left(2\sqrt{2},\frac{2}{3}\right)$

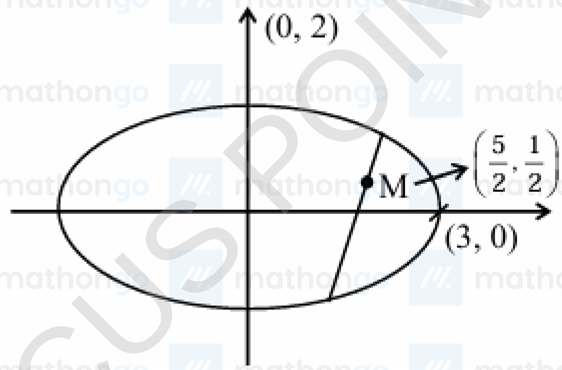
Length of chord $=\sqrt{(2\sqrt{2})^2+\left(\frac{2}{3}-2\right)^2}$

$$=\sqrt{8+\frac{16}{9}}$$
$$=\frac{\sqrt{88}}{3}=\frac{2\sqrt{22}}{3}$$

On comparing $\alpha=22$

Q6.

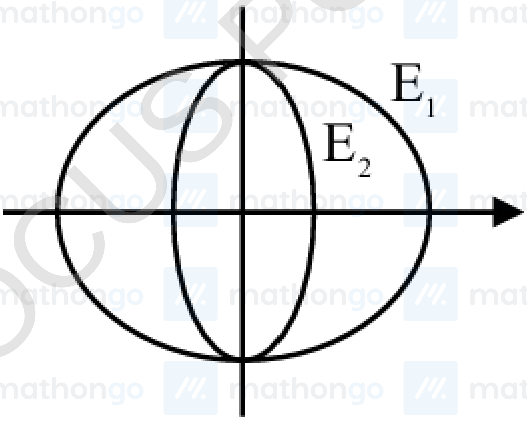
(1)



Equation of chord $\mathrm{T}=\mathrm{S}_1$

$$\frac{5}{2}\left(\frac{x}{9}\right)+\frac{1}{2}\left(\frac{y}{4}\right)=\frac{25}{36}+\frac{1}{16}$$
$$\Rightarrow \frac{5x}{18}+\frac{y}{8}=\frac{100+9}{144}=\frac{109}{144}$$
$$\Rightarrow 40x+18y=109$$
$$\Rightarrow \alpha=40,\beta=18$$
$$\Rightarrow \alpha+\beta=58$$

Q7.
(54)



$$E_1 = \frac{x^2}{9} + \frac{y^2}{4} \Rightarrow e = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$E_2 : \frac{x^2}{a^2} + \frac{y^2}{4} = 1$$

$$e = \frac{\sqrt{5}}{3} = \sqrt{1 - \frac{a^2}{4}} \Rightarrow \frac{5}{9} = 1 - \frac{a^2}{4}$$

$$a^2 = \frac{16}{9}$$

$$E_2 : \frac{x^2}{\frac{16}{9}} + \frac{y^2}{4} = 1$$

$$E_3 : \frac{x^2}{\frac{16}{9}} + \frac{y^2}{b^2} = 1$$

$$e = \frac{\sqrt{5}}{3} = \sqrt{1 - \frac{b^2}{\frac{16}{9}}} \Rightarrow b^2 = \frac{64}{81}$$

$$E_3 = \frac{x^2}{\frac{16}{9}} + \frac{y^2}{\frac{64}{81}} = 1$$

$$A_1 = \pi \times 3 \times 2 \Rightarrow 6\pi$$

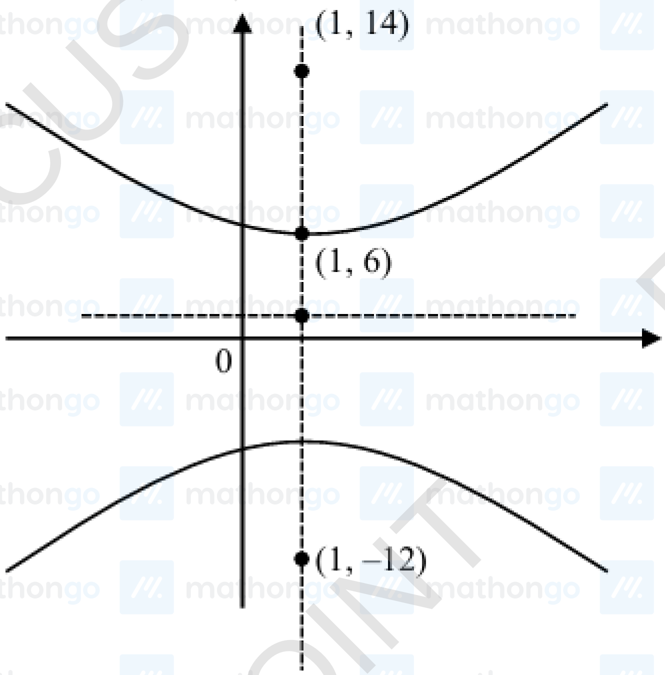
$$A_2 = \pi \times \frac{4}{3} \times 2 = \frac{8\pi}{3}$$

$$A_3 = \pi \times \frac{4}{3} \times \frac{8}{9} = \frac{32\pi}{81}$$

$$\sum_{i=1}^{\infty} A_i = 6\pi + \frac{8\pi}{3} + \frac{32\pi}{81} + \ldots .\infty \Rightarrow \frac{6\pi}{1 - \frac{4}{9}} \Rightarrow \frac{54\pi}{5}$$

$$\therefore \frac{5}{\pi} \sum_{i=1}^{\infty} A_i \Rightarrow \frac{5}{\pi} \times \frac{54\pi}{5} = 54$$

Q1.
(4)



$\mathrm{be}=13,\ \mathrm{b}=5$
$\mathrm{a}^2=\mathrm{b}^2\left(\mathrm{e}^2-1\right)$
$=\mathrm{b}^2\mathrm{e}^2-\mathrm{b}^2$
$=169-25=144$
$\ell(\mathrm{LR})=\frac{2\mathrm{a}^2}{\mathrm{b}}=\frac{2\times 144}{5}=\frac{288}{5}$

Q2. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
(55) $\frac{2b^2}{a}=15\sqrt{2}$ ...(i)
$\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{5}{2}}$....(ii)
From (i) and (ii)
$a=5\sqrt{2}$ and $b^2=75$
$\frac{x^2}{A^2}-\frac{y^2}{B^2}=-1$
$\frac{2A^2}{B}=12\sqrt{5}$ ....(iii)
Since, product of transverse axis is $=100\sqrt{10}$
$(2A)\cdot(2B)=100\sqrt{10}$
From (iii) and (iv)
$A^2=150$ and $B=5\sqrt{5}$
$e_2=\sqrt{1+\frac{A^2}{B^2}}=\sqrt{\frac{11}{5}}$
$\therefore 25e_2^2=25\left(\frac{11}{5}\right)=55$

Q3. C: $x^2+y^2-8x=0$ H : $\frac{x^2}{9}-\frac{y^2}{4}=1$

(3) By solving $\frac{x^2}{9}-\left(\frac{8x-x^2}{4}\right)=1$

$4x^2-72x+9x^2=36$

$\Rightarrow 13x^2-72x-36=0$

$\Rightarrow 13x^2-78x+6x-36=0$

$\Rightarrow 13x(x-6)+6(x-6)=0$

$\Rightarrow x=6$ or $-\dfrac{13}{6}\times$ neglected

$\Rightarrow y^2=8(6)-(6)^2$

$\Rightarrow y=\pm\sqrt{12}$

So, points A and B are $(6,\sqrt{12}),(6,-\sqrt{12})$

$P\left(h,\frac{2h+4}{3}\right)$

Centroid of $\triangle PAB$ is $\left(\frac{12+h}{3},\frac{2h+4}{9}\right)$

By options this centroid lies on the live $6x-9y=20$

Q4. $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ foci are $(ae,0)$ and $(-ae,0)$

(3) $\dfrac{x^2}{A^2}+\dfrac{y^2}{B^2}=1$ foci are $\left(Ae',0\right)$ and $\left(-Ae',0\right)$

$\Rightarrow 2ae=2\sqrt{3}\Rightarrow ae=\sqrt{3}$

and $2Ae'=2\sqrt{3}\Rightarrow Ae'=\sqrt{3}$

$\Rightarrow ae=Ae'\Rightarrow \dfrac{e}{e'}=\dfrac{A}{a}$

$\Rightarrow \dfrac{1}{3}=\dfrac{A}{a}\Rightarrow a=3A$

Now $a-A=2\Rightarrow a-\frac{a}{3}-2\Rightarrow a=3$ and $A=1$

$Ae=\sqrt{3}\Rightarrow e=\frac{1}{\sqrt{3}}$ and $e'=\sqrt{3}$

$b^2=a^2\left(1-e^2\right)$

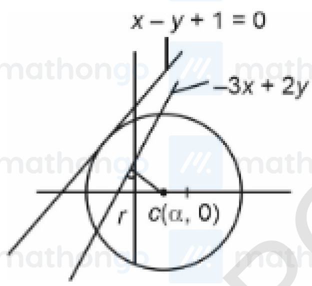
$b^2=6$

and $B^2=A^2\left((e')^2-1\right)=(2)\Rightarrow B^2=2$

sum of LR $=\frac{2b^2}{a}+\frac{2B^2}{A}=8$

Q5.

(19)



$r=\left|\frac{a+1}{\sqrt{2}}\right|\Rightarrow (a+1)^2=2r^2$

Also $\left(\frac{3a-1}{\sqrt{13}}\right)^2+\left(\frac{2}{\sqrt{13}}\right)^2=r^2$

$$\Rightarrow\left(\frac{3a-1}{\sqrt{13}}\right)^2+\frac{4}{13}=\frac{(a+1)^2}{2}$$

$$5a^2-14a-3=0$$

$\therefore \quad a=-\frac{1}{5},3$

$\because \quad a\neq-\frac{1}{5}\Rightarrow$

$\Rightarrow \quad r=2\sqrt{2}$

$\because$ One focus of $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ is $(3,0)$

$\Rightarrow \alpha e=3$ and $2\alpha=4\sqrt{2}$

$\Rightarrow \alpha=2\sqrt{2}\Rightarrow\alpha^2=8$

$$\alpha^2\left[1+\frac{\beta^2}{\alpha^2}\right]=9$$

$\alpha^2+\beta^2=9$

$\Rightarrow \beta^2=1$

$\therefore 2\alpha^2+3\beta^2=19$

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Q1. $\sin x + \sin^2 x = 1$

(4) $\Rightarrow \sin x = \cos^2 x \Rightarrow \tan x = \cos x$

$\therefore$ Given expression

$= 2\cos^{12} x + 6\left[\cos^{10} x + \cos^8 x\right] + 2\cos^6 x$

$= 2\left[\sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x\right]$

$= 2\sin^3 x\left[(\sin x + 1)^3\right]$

$= 2\left[\sin^2 x + \sin x\right]^3$

$= 2$

Q2. $(\sin 70^\circ)(\cot 10^\circ \cot 70^\circ - 1)$

(2) $= \sin 70^\circ \cot 10^\circ \cot 70^\circ - \sin 70^\circ$

$= \cot 10^\circ \cos 70^\circ - \sin 70^\circ$

$= \dfrac{\cos 10^\circ \cos 70^\circ - \sin 70^\circ \sin 10^\circ}{\sin 10^\circ}$

$= \dfrac{\cos(10^\circ + 70^\circ)}{\sin 10^\circ}$

$= \dfrac{\cos 80^\circ}{\sin 10^\circ} = 1$

Q3. $\dfrac{12}{13}\cos x + \dfrac{5}{13}\sin x$

(3) Let $\tan\alpha = \dfrac{5}{12}, \alpha \in \left(0, \dfrac{\pi}{2}\right)$

$\Rightarrow \sin\alpha = \dfrac{5}{13}, \cos\alpha = \dfrac{12}{13}$

$\Rightarrow \dfrac{12}{13}\cos x + \dfrac{5}{13}\sin x = \cos\alpha\cos x + \sin\alpha\sin x$

$= \cos(x - \alpha)$

$\Rightarrow \cos^{-1}[\cos(x - \alpha)] = x - \alpha$

$= x - \tan^{-1}\left(\dfrac{5}{12}\right)$

Q4. $\frac{1}{\sin\frac{\pi}{6}}\sum_{r=1}^{13}\frac{\sin\left[\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)-\left(\frac{\pi}{4}\right)-(r-1)\frac{\pi}{6}\right]}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}$

(4)

$\dfrac{1}{\sin\frac{\pi}{6}}\sum_{\mathrm{r}=1}^{13}\left(\cot\left(\dfrac{\pi}{4} + (\mathrm{r}-1)\dfrac{\pi}{6}\right) - \cot\left(\dfrac{\pi}{4} + \dfrac{\mathrm{r}\pi}{6}\right)\right)$

$= 2\sqrt{3} - 2 = \alpha\sqrt{3} + \mathrm{b}$

So $a^2 + b^2 = 8$

Q1. $2\sin^2\theta = \cos 2\theta$

(3) $2\sin^2\theta = 1 - 2\sin^2\theta$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$2\cos^2\theta = 3\sin\theta$

$2 - 2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2\sin\theta - 1)(2\sin\theta - 2) = 0$

$\sin\theta = \frac{1}{2}$

so common equation which satisfy both equations is $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad (\theta \in [0, 2\pi])$

$\text{Sum} = \pi$

Q1. (14)

Let $\tan^{-1}\alpha = A \Rightarrow \tan A = \alpha$

$\cot^{-1}\beta = B \Rightarrow \cot B = \beta$

$\sec^2 A + \operatorname{cosec}^2 B = 36$

$\Rightarrow 1 + \tan^2 A + 1 + \cot^2 B = 36$

$\Rightarrow \alpha^2 + \beta^2 = 34$

Also $\alpha + \beta = 8$ (Given)

$\therefore (\alpha+\beta)^2 = 34 + 2\alpha\beta = 64$

$\Rightarrow \alpha\beta = 15$

$\Rightarrow \alpha, \beta$ are roots of equation

$x^2 - 8x + 15 = 0$

$\Rightarrow (x-3)(x-5) = 0$

$\Rightarrow \quad x = 3, 5$

$\therefore \quad \alpha = 3, \beta = 5 \quad (\alpha < \beta)$

$\therefore \quad \alpha^2 + \beta = 9 + 5 = 14$

Q2. (2)

$$\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$$

$$\cos\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{33}{56}\right)$$

$$\cos\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1+\frac{3}{4}\cdot\frac{5}{12}}\right) + \tan^{-1}\frac{33}{56}\right)$$

$$\cos\left(\tan^{-1}\frac{56}{33} + \cot^{-1}\frac{56}{33}\right)$$

$$\cos\left(\frac{\pi}{2}\right) = 0$$

Q3. (2)

$16\left(\sec^{-1}x\right)^2 + \left(\operatorname{cosec}^{-1}x\right)^2$

$\operatorname{Sec}^{-1}x = a \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\operatorname{cosec}^{-1}x = \frac{\pi}{2} - a$

$$= 16\left[a^2 + \left(\frac{\pi}{2} - a\right)^2\right] = 16\left[2a^2 - \pi a + \frac{\pi^2}{4}\right]$$

$$\max]_{a=\pi} = 16\left[2\pi^2 - \pi^2 + \pi\frac{2}{4}\right] = 20\pi^2$$

$$\min]_{a=\frac{\pi}{4}} = 16\left[\frac{2\times\pi^2}{16} - \frac{\pi^2}{4} + \frac{\pi^2}{4}\right] = 2\pi^2$$

Sum $= 22\pi^2$

Q4. (1)

$$\Rightarrow \cot^{-1}\left(\frac{\alpha\beta+1}{\alpha-\beta}\right) + \cot^{-1}\left(\frac{\beta\gamma+1}{\beta-\gamma}\right) + \cot^{-1}\left(\frac{\alpha\gamma+1}{\gamma-\alpha}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right) + \tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right) + \pi + \tan^{-1}\left(\frac{\gamma-\alpha}{1+\gamma\alpha}\right)$$

$$\Rightarrow \left(\tan^{-1}\alpha - \tan^{-1}\beta\right) + \left(\tan^{-1}\beta - \tan^{-1}\gamma\right) + \left(\pi + \tan^{-1}\gamma - \tan^{-1}\alpha\right)$$

$\Rightarrow \pi$

Q5. $\cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x+1)$

(5) $2\cos^{-1} x - \sin^{-1}(2x+1) = \frac{3\pi}{2}$

$2\alpha - \beta = \frac{3\pi}{2}$ where $\cos^{-1} x = \alpha, \sin^{-1}(2x+1) = \beta$

$2\alpha = \frac{3\pi}{2} + \beta$

$\cos 2\alpha = \sin\beta$

$2\cos^2\alpha - 1 = \sin\beta$

$2x^2 - 1 = 2x + 1$

$x^2 - x - 1 = 0$

$\Rightarrow \mathrm{n} = \frac{1 \pm \sqrt{5}}{2} = \left[\begin{array}{l} \mathrm{n} = \frac{1+\sqrt{5}}{2} \text{ rejedcted} \\ \quad \mathrm{n} = \frac{1-\sqrt{5}}{2} \end{array}\right.$

$\therefore 4\mathrm{x}^2 - 4\mathrm{x} = 4$

$(2\mathrm{x} - 1)^2 = 5$

Q1. $\vec{b} = \vec{a} \times (\hat{i} - 3\hat{k})$

(1)

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = 2\hat{i} + 8\hat{j} + \hat{k}$$

$\vec{c} = \vec{b} \times \hat{k} = 8\hat{i} - 2\hat{j}$

$\vec{c} - 2\hat{j} = 8\hat{i} - 4\hat{j}$

Projection of $(\hat{i} - 2\hat{j})$ on $\vec{a}$

$$(\vec{c} - 2\hat{j}) \cdot \hat{a} = \frac{\langle 8, -4, 0\rangle \cdot \langle 3, -1, 2\rangle}{\sqrt{14}}$$

$$= \frac{28}{\sqrt{14}} = 2\sqrt{14}$$

Q2. let

(1) $\vec{a}_{11} =$ component of $\vec{a}$ along $\vec{b}$

$\vec{a}_1 =$ component of $\vec{a}$ perpendicular to $\vec{b}$

$\vec{a}_{11} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$

$\vec{a}_1 = \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$

$\because \vec{a} = \vec{a}_{11} + \vec{a}_1$

$\therefore \vec{a} =$

$= \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k}) + \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$

$\vec{11}\hat{j} - \frac{33}{11}\hat{k}$

$\alpha = 4\hat{i} + \hat{j} - 3\hat{k}$

$\alpha^2 + \beta^2 + \gamma^2 = 16 + 1 + 9 = 26$

Q3. $\hat{a} \cdot \hat{b} = \frac{1}{2}$

(3)

Now $(\lambda\hat{a} + 2\hat{b}) \cdot (3\hat{a} - \lambda\hat{b}) = 0$

$3\lambda\hat{a} \cdot \hat{a} - \lambda^2\hat{a} \cdot \hat{b} + 6\hat{a} \cdot \hat{b} - 2\lambda\hat{b} \cdot \hat{b} = 0$

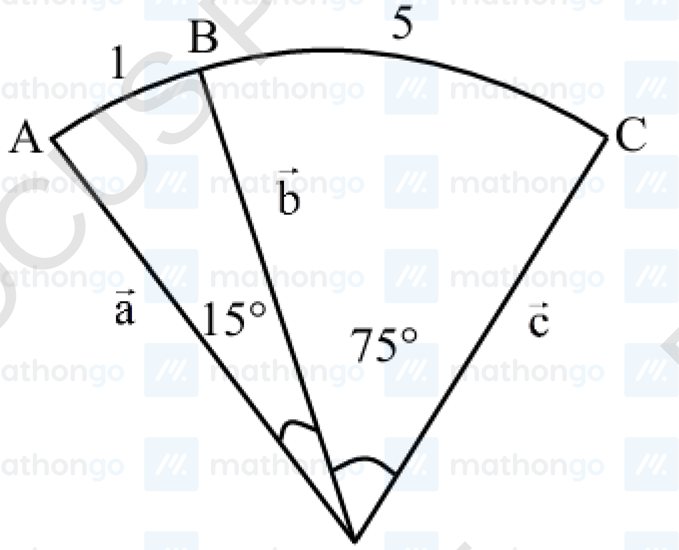
$3\lambda - \frac{\lambda^2}{2} + 3 - 2\lambda = 0$

$\lambda^2 - 2\lambda - 6 = 0$

$\lambda = 1 \pm \sqrt{7}$

$\Rightarrow$ number of values $= 0$

Q4.
(2)



$\vec{c} = \alpha \vec{a} + \beta \vec{b} \ldots\ldots (1)$

$\vec{a} \cdot \vec{c} = \alpha \vec{a} \cdot \vec{a} + \beta \vec{b} \cdot \vec{a}$

$0 = \alpha + \beta \cos 15^\circ \ldots\ldots (2)$

$(1) \Rightarrow \vec{b} \cdot \vec{c} = \alpha \vec{a} \cdot \vec{b} + \beta \vec{b} \cdot \vec{b}$

$\Rightarrow \cos 75^\circ = \alpha \cos 15^\circ + \beta \ldots\ldots (3)$

$(2)\&(3) \Rightarrow \cos 75^\circ = -\beta \cos^2 15^\circ + \beta$

$\beta = \frac{\cos 75^\circ}{\sin^2 15^\circ} = \frac{1}{\sin 15^\circ} = \frac{2\sqrt{2}}{\sqrt{3}-1}$

$(2) \Rightarrow \alpha = \frac{-\cos 15^\circ}{\sin 15^\circ} = \frac{-(\sqrt{3}+1)}{(\sqrt{3}-1)}$

$\therefore \vec{c} = \frac{-(\sqrt{3}+1)}{(\sqrt{3}-1)} \vec{a} + \left(\frac{2\sqrt{2}}{\sqrt{3}-1}\right) \vec{b}$

Now

$$\alpha + \sqrt{2}(\sqrt{3}-1)\beta = \frac{-(\sqrt{3}+1)}{(\sqrt{3}-1)} + \frac{\sqrt{2}(\sqrt{3}-1) \cdot 2\sqrt{2}}{\sqrt{3}-1}$$

$$= \frac{-(\sqrt{3}+1)^2}{2} + 4$$

$$= \frac{-3-1-2\sqrt{3}+8}{2}$$

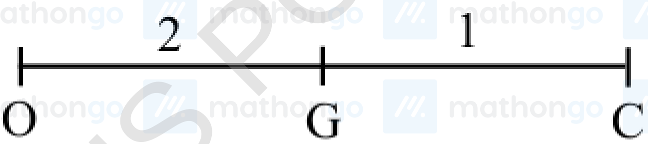
$$= 2-\sqrt{3}$$

Q5. $\vec{c} = \lambda(\vec{b} \times (\vec{a} \times \vec{b}))$
(1)
$= \lambda((\vec{b} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{b})$
$= \lambda(11\vec{a} - 2\vec{b}) = \lambda(11i + 22j + 33k - 6i - 2j + 2k)$
$= \lambda(5i + 20j + 35k)$
$= 5\lambda(5i + 4j + 7k)$
$=$ Given $\vec{c} \cdot \vec{a} = 5$
$= 5\lambda(1 + 8 + 21) = 5 \quad = \lambda = \dfrac{1}{30}$
$\Rightarrow \vec{c} = \dfrac{1}{6}(i + 4j + 7k)$
$\vec{c} = \dfrac{\sqrt{1 + 16 + 49}}{6} = \sqrt{\dfrac{11}{6}}$

Q6. Equation of line in the internal bisector of $O\dot{A}$ and OB is $(\sqrt{3}+1)\hat{i} + (\sqrt{3}+1)\hat{j}$
(3) $\Rightarrow$ line will be $y = x \Rightarrow x - y = 0$
$D = \left|\frac{a-(1-a)}{\sqrt{a^2+(1-a)^2}}\right| = \frac{9}{\sqrt{2}}$
$(2a-1)^2 = \dfrac{81}{2}\left(a^2 + (1-a)^2\right)$
$\Rightarrow 2\left(4a^2 - 4a + 1\right) = 81a^2 + 81a^2 - 162a - 81$
$\Rightarrow 162a^2 - 162a + 81 - 8a^2 + 8a - 2 = 0$
$\Rightarrow 154a^2 - 154a + 79 = 0$
Sum of values $= -\dfrac{(-154)}{154} = 1$

Q7. $\vec{c} = \left(\frac{\vec{b}\cdot\vec{a}}{|\vec{b}|}\right)\frac{\vec{a}}{|\vec{a}|}$
(16)
$= \left(\dfrac{\lambda+8}{9}\right)(\hat{i} + 2\hat{j} + 2\hat{k})$
$|\vec{a} + \vec{c}| = 7$
$\Rightarrow \left(\dfrac{\lambda+8}{9} + 1\right)\hat{i} + \left(\dfrac{2(\lambda+8)}{9} + 2\right)\hat{j} + \left(\dfrac{2(\lambda+8)}{9} + 2\right)\hat{k}\Bigg| = 7$
$\left(\dfrac{\lambda+8}{9} + 1\right)^2 + \left(\dfrac{2(\lambda+8)}{9} + 2\right)^2 + \left(\dfrac{2(\lambda+8)}{9} + 2\right)^2 = 49$
$\Rightarrow \lambda = 4 \Rightarrow \vec{c} = \frac{4}{3}\hat{i} + \frac{8}{3}\hat{j} + \frac{8}{3}\hat{k}$
Area of parallelogram $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \\ 4 & 0 & 4 \end{vmatrix} = 16$

Q8. We know that
(1)



$O\left((\text{orthocentre}) \frac{\overrightarrow{p}+\overrightarrow{q}+\overrightarrow{r}}{4}\right.$

$C(\text{ circum centre}) \, \alpha\overrightarrow{p}+\beta\overrightarrow{q}+\gamma\overrightarrow{r}$

$C\,(\text{centroid}) = \frac{\overrightarrow{p}+\overrightarrow{q}+\overrightarrow{r}}{3}$

by relation

$\Rightarrow 2(\alpha\overrightarrow{p}+\beta\overrightarrow{q}+\gamma\overrightarrow{r})+\frac{\overrightarrow{p}+\overrightarrow{q}+\overrightarrow{r}}{4}=3\left(\frac{\overrightarrow{p}+\overrightarrow{q}+\overrightarrow{r}}{3}\right)$

$\Rightarrow 8(\alpha\overrightarrow{p}+\beta\overrightarrow{q}+\gamma\overrightarrow{r})=3(\overrightarrow{p}+\overrightarrow{q}+\overrightarrow{r})$

$\Rightarrow 8\alpha=3, 8\beta=3, 8\gamma=3$

$\alpha=\frac{3}{8}, \beta=\frac{3}{8}, \gamma=\frac{3}{8}$

$\therefore \alpha+2\beta+3\gamma$

$\frac{3}{8}+\frac{6}{8}+\frac{15}{8}=\frac{24}{8}=3$

Q9. $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

(6) $\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$

$\vec{d}=\vec{a}\times\vec{b}$

$=-\hat{i}+\hat{j}$

$|\vec{c}-2\vec{a}|^2=8$

$|\vec{c}|^2+4|\vec{a}|^2-4\vec{a}\cdot\vec{c}=8$

$|\vec{c}|^2+12-4|\vec{c}|=8$

$|\vec{c}|^2-4|\vec{c}|+4=0$

$|\vec{c}|^2=2$

$\vec{d}=\vec{a}\times\vec{b}$

$\vec{d}\times\vec{c}=(\vec{a}\times\vec{b})\times\vec{c}$

$\left(|\vec{d}|\times|\vec{c}|\sin\frac{\pi}{4}\right)^2=((\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a})^2$

$4=4|\vec{b}|^2+(\vec{b}\cdot\vec{c})2\left(|\vec{a}|^2\right)-2(\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{b})$

Let $\vec{b}\cdot\vec{c}=x$

$4=36+3x^2-20x$

$3x^2-20x+32=0$

$x=\frac{8}{3}, 4$

$\Rightarrow \vec{b}\cdot\vec{c}=\frac{8}{3}, 4$

$\Rightarrow \vec{b}\cdot\vec{c}=\frac{8}{3}$

Now, $|10-3\vec{b}\cdot\vec{c}|+|\vec{d}\times\vec{c}|^2$

$=|10-8|+(2)^2=6$

Q10. $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$
(4) $\vec{b} = 3\hat{i} - 5\hat{j} + 3\hat{k}$

$\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$

$\vec{a} \times \vec{c} + \vec{b} \times \vec{c} = 0$

$(\vec{a} + \vec{b}) \times \vec{c} = 0$

$\Rightarrow \vec{c} = \lambda(\vec{a} + \vec{b})$

$\vec{c} = \lambda(5\hat{i} - 6\hat{j} + 4\hat{k}) \ldots\ldots (1)$

$|\vec{c}|^2 = \lambda^2(25 + 36 + 16)$

$|\vec{c}|^2 = 77\lambda^2$

$(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 168$

$\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{c} \cdot \vec{b} + |\vec{c}|^2 = 168$

$14 + \vec{c} \cdot (\vec{a} + \vec{b}) + 77\lambda^2 = 168$

using equation (1)

$\lambda|5\hat{i} - 6\hat{j} + 4\hat{k}|^2 + 77\lambda^2 = 154$

$77\lambda + 77\lambda^2 - 154 = 0$

$\lambda^2 + \lambda - 2 = 0$

$\lambda = -2, 1$

$\therefore$ Maximum value of $|\vec{c}|^2$ occurs when $\lambda = -2$

$|\vec{c}|^2 = 77\lambda^2$

$= 77 \times 4$

$= 308$

Q11.
(2)
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(-7) + 7\hat{j} + 7\hat{k}$$

$$\hat{a} = \pm\frac{(-7\hat{i} + 7\hat{j} + 7\hat{k})}{\sqrt{7^2 + 7^2 + 7^2}} = \pm\left(\frac{-\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right)$$

$$\text{Now, } \cos\theta = \pm\frac{(-1 + 1 + 1)}{\sqrt{3} \cdot \sqrt{3}} = \pm\frac{1}{3}$$

$$\Rightarrow \cos^{-1}\left(\frac{-1}{3}\right) \Rightarrow \hat{a} = \frac{-(-\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}$$

$$\hat{a} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$$

$\cos\frac{\pi}{3} = \frac{1-\alpha-1}{\sqrt{3}\cdot\sqrt{\alpha^2+2}}$

$\frac{1}{2} = \frac{-\alpha}{\sqrt{3}\cdot\sqrt{\alpha^2+2}} \rightarrow \alpha < 0$

$3\left(\alpha^2+2\right) = 4\alpha^2$

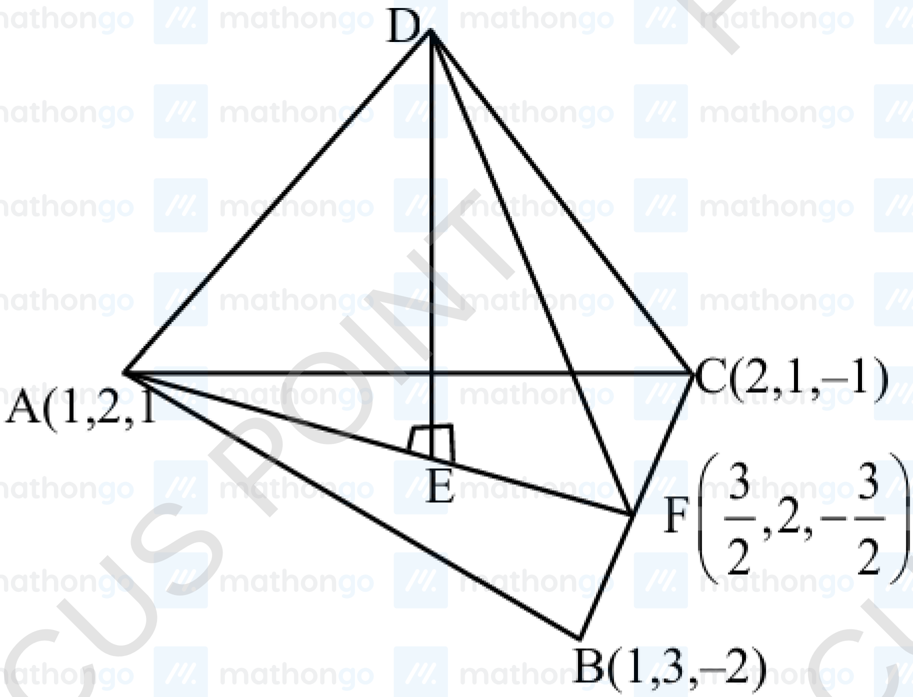
$6 = \alpha^2$

$\alpha = \pm\sqrt{6}$

Clearly, $\alpha = -\sqrt{6}$

Q12.

(4)



Area of $\triangle ABC = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$= \frac{1}{2}|5\hat{i}+3\hat{j}+\hat{k}| = \frac{1}{2}\sqrt{35}$

volume of tetrahedron

$= \frac{1}{3}\times \text{ Base area } \times h = \frac{\sqrt{805}}{6\sqrt{2}}$

$\frac{1}{3}\times\frac{1}{2}\sqrt{35}\times h = \frac{\sqrt{805}}{6\sqrt{2}}$

$h = \sqrt{\frac{23}{2}}$

$$\mathrm{AE}^2 = \mathrm{AD}^2 - \mathrm{DE}^2 = \tfrac{13}{18} \therefore \mathrm{AE} = \sqrt{\tfrac{13}{18}}$$

$$\overrightarrow{\mathrm{AE}} = |\mathrm{AE}| \cdot \left(\frac{\hat{\mathrm{i}} - 5\hat{\mathrm{k}}}{\sqrt{26}}\right)$$

$$= \sqrt{\frac{13}{18}} \cdot \left(\frac{\hat{\mathrm{i}} - 5\hat{\mathrm{k}}}{\sqrt{26}}\right)$$

$$= \sqrt{\frac{13}{18}} \cdot \left(\frac{\hat{\mathrm{i}} - 5\hat{\mathrm{k}}}{\sqrt{26}}\right) = \frac{\hat{\mathrm{i}} - 5\hat{\mathrm{k}}}{6}$$

$$\text{P.V. of E} = \frac{\hat{\mathrm{i}} - 5\hat{\mathrm{k}}}{6} + \hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}} = \frac{1}{6}(7\hat{\mathrm{i}} + 12\hat{\mathrm{j}} + \hat{\mathrm{k}})$$

Q1. Point B

(216) $(3\lambda+1,-\lambda+1,-1) \equiv (2\mu+2,0,\alpha\mu-4)$

$3\lambda+1=2\mu+2$

$-\lambda+1=0$

$-1=\alpha\mu-4$

$\lambda=1, \mu=1, \alpha=3$

$B(4,0,-1)$

Let Point ' P ' is $(2\delta+2,0,3\delta-4)$

Dr's of AP $<2\delta+1,-1,3\delta-3>$

$AP \perp L_2 \Rightarrow \delta=\frac{7}{13}$

$P\left(\frac{40}{13}, 0, \frac{-31}{13}\right)$

$\therefore 26\alpha(PB)^2=26\times3\times\left(\frac{144}{169}+\frac{324}{169}\right)$

$=216$

Q2. Line passing through $(1,4,0)$ and parallel to $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ is $L: \frac{x-1}{1}=\frac{y-4}{2}=\frac{z}{3}$

(3) Any point on $L:(\lambda+1,2\lambda+4,3\lambda)$

Any point on $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ is $(2\mu+2,3\mu+6,4\mu+3)$

$$\left.\begin{array}{l}\lambda+1=2\mu+2\\2\lambda+4=3\mu+6\\3\lambda=4\mu+3\end{array}\right\}\lambda=1\mu=0$$

Point: $(2,6,3)$

Distance $=\sqrt{(2-1)^2+(6-4)^2+(3-0)^2}$

$=\sqrt{1+4+9}=\sqrt{14}$

Q3. $\vec{a}=(2,1,-3)$

(2) $\vec{b}=(-1,-3,-5)$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\1 & 2 & -3\\2 & 4 & -5\end{vmatrix}$$

$=2\hat{i}-\hat{j}$

$\vec{b}-\vec{a}=-3\hat{i}-4\hat{j}-2\hat{k}$

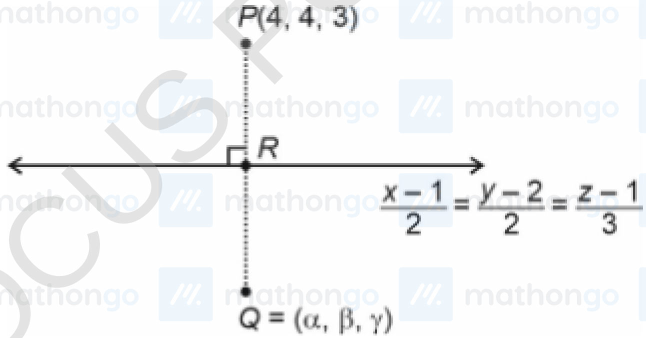
$$S_d=\frac{|(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}$$

$=\frac{2}{\sqrt{5}}$

$(S_d)^2=\frac{4}{5}$

$m=4, n=5 \Rightarrow m+n=9$

Q4.
(1)



Let coordinate of $R=(2r+1, r+2, 3r+1)$

$\because \quad PR$ is perpendicular to given line.

$\therefore (2r-3)\cdot 2+(r-2)\cdot 1+(3r-2)\cdot 3=0$

$\therefore r=1$

$\therefore$ Coordinate of $R=(3,3,4)$

$\therefore \quad (\alpha, \beta, \gamma)=(2,2,5)$

$\therefore \alpha+\beta+\gamma=9$

Q5.
(3)

DR's of $L_3=\overrightarrow{\mathrm{m}}\times\overrightarrow{\mathrm{n}}=\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}\\ 1 & -1 & 2\\ -1 & 2 & 1\end{vmatrix}$

$=-5\hat{\mathrm{i}}-3\hat{\mathrm{j}}+\hat{\mathrm{k}}$

$L_3: \frac{x-\alpha}{-5}=\frac{y-\beta}{-3}=\frac{z-\gamma}{1}=\lambda$

$A(\alpha-5\lambda, \beta-3\lambda, \gamma+\lambda)$

$L_1: \frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-1}{2}=k$

$\mathrm{B}(\mathrm{k}+1,-\mathrm{k}+2,2\mathrm{k}+1)$

Now

$\alpha-5\lambda=\mathrm{k}+1 \Rightarrow \alpha=5\lambda+\mathrm{k}+1$

$\beta-3\lambda=-\mathrm{k}+2 \Rightarrow \beta=3\lambda-\mathrm{k}+2$

$\gamma+\lambda=2\mathrm{k}-1 \Rightarrow \gamma=-\lambda+2\mathrm{k}+1$

$|5\alpha-11\beta-8\gamma|=|-25|$

$=25$

Q6.
(4)

Vector parallel to L

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & 3 & 4\end{vmatrix}=10\hat{i}-10\hat{j}+5\hat{k}$

$=5(2\hat{i}-2\hat{j}+\hat{k})$

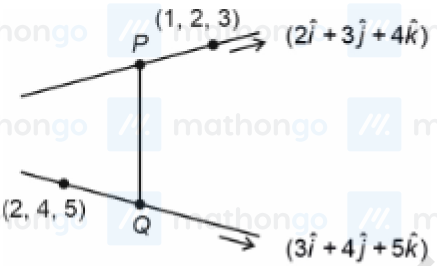
Equation of ' L '

$\frac{x-2}{2}=\frac{y+1}{-2}=\frac{z-3}{1}=\lambda(\text{ say })$

Let $Q(2\lambda+2,-2\lambda-1,\lambda+3)$
$\Rightarrow \quad 2\lambda+2=0 \Rightarrow \lambda=-1$
$\Rightarrow Q(0,1,2)$
$d(P,Q)=3$

Q7. $\mathrm{A}(\mathrm{x},\mathrm{y},\mathrm{z})$ Let $\mathrm{P}(0,3,2),\mathrm{Q}(2,0,3),\mathrm{R}(0,0,1)$
(3) $\mathrm{AP}=\mathrm{AQ}=\mathrm{AR}$
$\mathrm{x}^2+(\mathrm{y}-3)^2+(\mathrm{z}-2)^2=(\mathrm{x}-2)^2+\mathrm{y}^2+(\mathrm{z}-3)^2=\mathrm{x}^2+\mathrm{y}^2+(\mathrm{z}-1)^2$
In xy plane $z=0$
So, $x^2-4x+4+y^2+9=x^2+y^2+1$
$x=3$
$9+y^2-6y+9+4=x^2+y^2+1$
So, $\mathrm{A}(3,2,0)$ also $\mathrm{B}(1,4,-1)\&\mathrm{C}(2,0,-2)$
Now $AB=\sqrt{4+4+1}=3$
$\mathrm{AC}=\sqrt{1+4+4}=3$
$\mathrm{BC}=\sqrt{1+16+1}=\sqrt{18}$
$\mathrm{AB}=\mathrm{AC}$
isosceles $\Delta \& \mathrm{AB}^2+\mathrm{AC}^2=\mathrm{BC}^2$
right angle Δ
Area of $\triangle\mathrm{ABC}=\frac{1}{2}\times$ base.height
$\frac{1}{2}\times 3\times 3=\frac{9}{2}$
So only S_1 is true

Q8. $L_1: \dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$
(1) $L_2: \dfrac{x-2}{3}=\dfrac{y-4}{4}=\dfrac{z-5}{5}$



$P(2\lambda+1,3\lambda+2,4\lambda+3)$
$Q(3\mu+2,4\mu+4,5\mu+5)$
Dr's of $PQ<2\lambda-3\mu-1,3\lambda-4\mu-2,$

$4\lambda - 5\mu - 2 >$

$PQ = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$

$\Rightarrow \frac{2\lambda - 3\mu - 1}{-1} = \frac{3\lambda - 4\mu - 2}{2} = \frac{4\lambda - 5\mu - 2}{-1}$

$\Rightarrow \lambda = \frac{1}{3}\mu = \frac{-1}{6}$

$\Rightarrow P\left(\frac{5}{3}, 3, \frac{13}{3}\right) \quad Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$

Dr's $PQ\langle 1, -2, 1\rangle$

$\therefore$ Line

$\frac{y-\frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{y-\frac{13}{3}}{1}$

Q9. Equation of line PQ is:

(2) $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = r(\text{ say })$

Let coordinate of $Q = (3r - 2, 2r - 1, 2r + 3)$

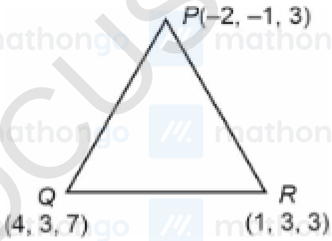
$\because PR = 5$

Then

$(3r - 2 - 1)^2 + (2r - 1 - 3)^2 + (2r + 3 - 3)^2 = 25$

$\therefore r = 0$ or 2

$\therefore$ Coordinate of $Q = (4, 3, 7)$

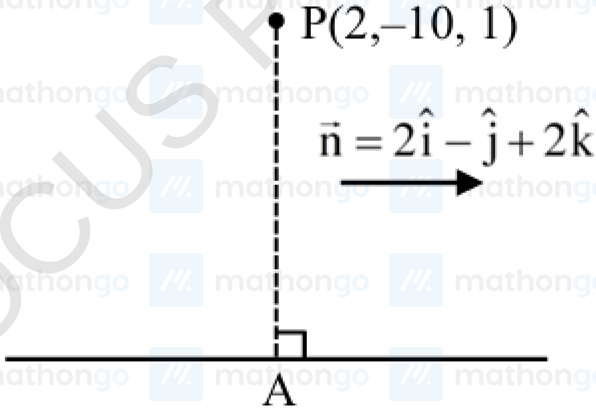


$\therefore$ square of area of $\triangle PQR = \left|\frac{1}{2}(\overrightarrow{PQ} \times \overrightarrow{PR})\right|^2$

$= \left|\frac{1}{2}(6\hat{i} + 4\hat{j} + 4\hat{k}) \times (3\hat{i} + 4\hat{j})\right|^2$

$= |-8\hat{i} + 6\hat{j} + 6\hat{k}|^2 = 136$

Q10.
(4)



$\left.\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2} = \lambda \text{ (let}\right)$

$(2\lambda + 1, -\lambda - 2, 2\lambda - 3)$

$\because \overrightarrow{PA} \cdot \vec{n} = 0$

$\Rightarrow (2\lambda - 1)2 + (-\lambda + 8)(-1) + (2\lambda - 4)2 = 0$

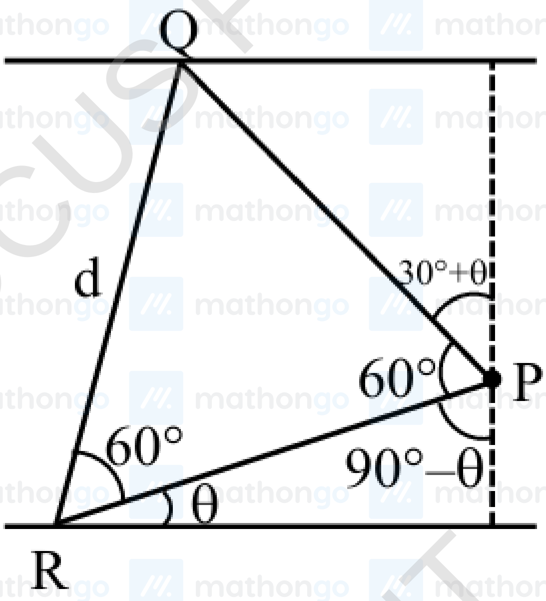
$\Rightarrow 4\lambda - 2 + \lambda - 8 + 4\lambda - 8 = 0$

$\Rightarrow 9\lambda - 18 = 0 \Rightarrow \lambda = 2$

$\therefore A(5, -4, 1)$

$\therefore AP = \sqrt{3^2 + 6^2 + 0^2} = \sqrt{45} = 3\sqrt{5}$

Q11.
(28)



$PR = \operatorname{cosec}\theta, PQ = 4\sec(30 + \theta)$

For equilateral

$d = PR = PQ$

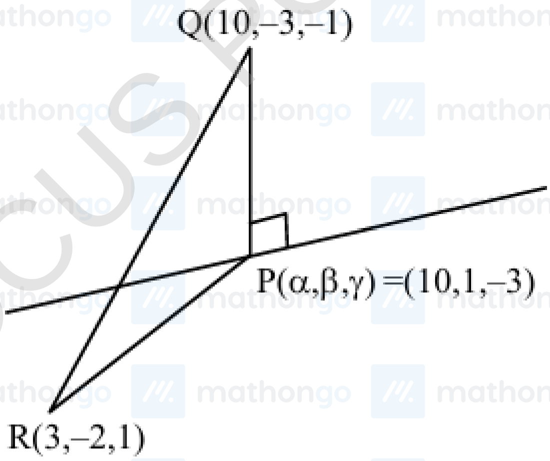
$\Rightarrow \cos(\theta + 30^\circ) = 4\sin\theta$

$\Rightarrow \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta = 4\sin\theta$

$\Rightarrow \tan\theta = \frac{1}{3\sqrt{3}}$

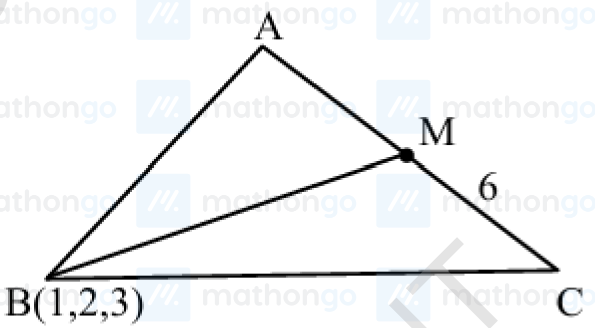
$QR^2 = d^2 = \operatorname{cosec}^2\theta = 28$

Q12.
(4)



$$\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}=\lambda$$
$\Rightarrow 7\lambda+3, -\lambda+2, -2\lambda-1$
dr's of QP $\Rightarrow$
$7\lambda-7, -\lambda+5, -2\lambda$
Now
$(7\lambda-7)\cdot 7-(-\lambda+5)+(2\lambda)\cdot 2=0$
$54\lambda-54=0 \Rightarrow \lambda=1$
$\therefore \mathrm{P}=(10,1,-3)$
$\overrightarrow{PQ}=-4\hat{j}+2\hat{k}$
$\overrightarrow{PR}=-7\hat{i}-3\hat{j}+4\hat{k}$
$$\text{Area}=\left|\frac{1}{2}\right|\begin{vmatrix} i & j & k \\ 0 & -4 & 2 \\ -7 & -3 & 4 \end{vmatrix}=3\sqrt{30}$$

Q13.
(2)



Let $M(3\lambda+6, 2\lambda+7, -2\lambda+7)$

$\overrightarrow{BM} = (3\lambda+5)\hat{i} + (2\lambda+5)\hat{j} + (-2\lambda+4)\hat{k}$

$\overrightarrow{AC} \cdot \overrightarrow{BM} = 0 = 3(3\lambda+5) + 2(2\lambda+5) - 2(-2\lambda+4)$

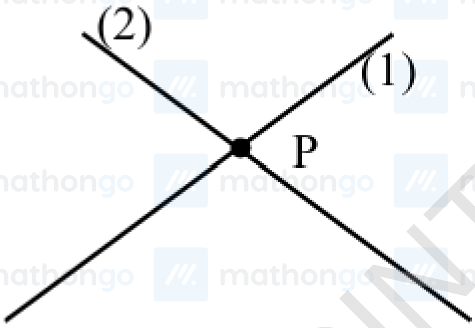
$\overrightarrow{BM} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$|\overrightarrow{BM} = 7$

Area $= \frac{1}{2} \times 6 \times 7 = 21$

Q14. Equation of line through point $(-1, 2, 1)$ is $\rightarrow$

(2)



$\Rightarrow \frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{4} - (2) = \lambda$

So, $\begin{bmatrix} x = 2\lambda - 1 \\ y = 3\lambda + 2 \\ z = 4\lambda + 1 \end{bmatrix}$

By(1) $\rightarrow \frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1} = \mu$(Let)

So, $\begin{bmatrix} x = 3\mu - 2 \\ y = 2\mu + 3 \\ z = \mu + 4 \end{bmatrix}$

For intersection point ' P '

$x = 2\lambda - 1 = 3\mu - 2$

$y = 3\lambda + 2 = 2\mu + 3$

$z = 4\lambda + 1 = \mu + 4$

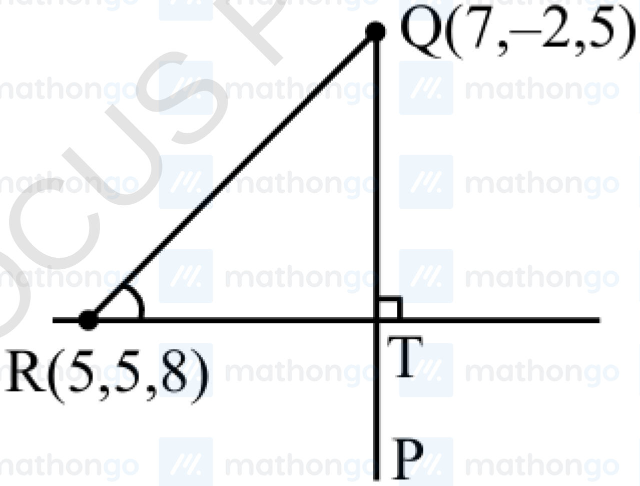
So, point $P(x, y, z) = (1, 5, 5)$

\& $Q(4, -5, 1)$

$\therefore PQ = \sqrt{9 + 100 + 16}$

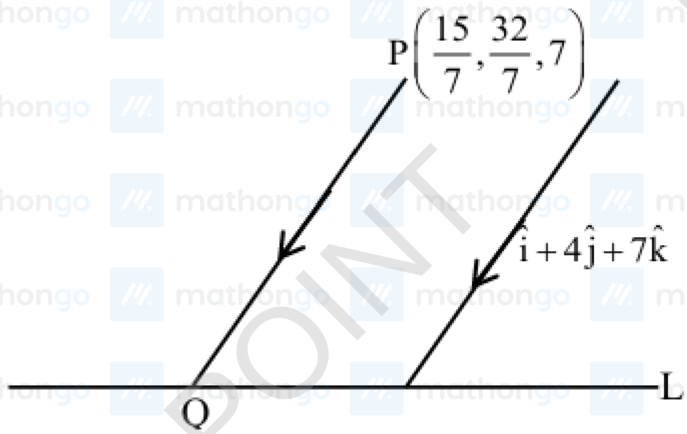
$= \sqrt{125} = 5\sqrt{5}$

Q15.
(957)



Let $R(2\lambda+1, 3\lambda-1, 4\lambda)$
$2\lambda+1=5$
$\lambda=2$
$R(5,5,8)$
let $T(2\lambda+1, 3\lambda-1, 4\lambda)$
$\overrightarrow{QT}=(2\lambda-6)\hat{i}+(3\lambda+1)\hat{j}+(4\lambda-5)\hat{k}$
$\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$
$\overrightarrow{QT}\cdot\vec{b}=0$
$4\lambda-12+9\lambda+3+16\lambda-20=0$
$\lambda=1$
$T(3,2,4)$
$QT=\sqrt{33} \quad RT=\sqrt{29}$
$(\text{ area of } \Delta PQR)^2=\left(\frac{1}{2}\sqrt{29}\cdot 2\sqrt{33}\right)^2$
$=957$

Q16.
(4)



$$L=\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$$

$$PQ=\frac{x-\frac{15}{7}}{1}=\frac{y-\frac{32}{7}}{4}=\frac{z-7}{7}=\lambda$$

$\Rightarrow Q\left(\lambda+\frac{15}{7}, 4\lambda+\frac{32}{7}, 7\lambda+7\right)$

Since Q lies on line L

So, $\frac{\lambda+\frac{15}{7}+1}{3}=\frac{7\lambda+7+5}{7}$

$\Rightarrow 7\lambda+22=21\lambda+36$

$\Rightarrow \lambda=-1$

$\therefore$ Point $Q\left(\frac{8}{7}, \frac{4}{7}, 0\right)$

$PQ=\sqrt{\left(\frac{15}{7}-\frac{8}{7}\right)^2+\left(\frac{32}{7}-\frac{4}{7}\right)^2+(7-0)}$

$PQ=\sqrt{66}$

$\Rightarrow (PQ)^2=66$

Q17.
(3)

$L_1: \overrightarrow{r}=(-\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}+2\hat{j}+\hat{k})$

$\Rightarrow \overrightarrow{r}=(\lambda-1)\hat{i}+2(\lambda+1)\hat{j}+(\lambda+1)\hat{k}$

$L_2: \overrightarrow{r}=(\hat{j}+\hat{k})+\mu(2\hat{i}+7\hat{j}+3\hat{k})$

$\Rightarrow \overrightarrow{r}=2\mu\hat{i}+(1+7\mu)\hat{j}+(1+3\mu)\hat{k}$

For point of intersection equating respective components

$\Rightarrow \lambda-1=2\mu$

$2(\lambda+1)=1+7\mu$

$\lambda+1=1+3\mu$

We get

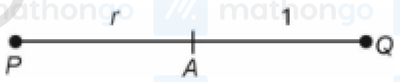
$\Rightarrow \lambda=3$ and $\mu=1$

$\Rightarrow \vec{a}+\vec{b}=3\hat{i}+9\hat{j}+4\hat{k}$

$L_3: \overrightarrow{r}=2\hat{i}+8\hat{j}+4\hat{k}+\alpha(3\hat{i}+9\hat{j}+4\hat{k})$

For $\alpha=2$, $\overrightarrow{r}=8\hat{i}+26\hat{j}+12\hat{k}$

Q18.
(4)



$A=\left(\frac{5r-1}{r+1},\frac{5r-1}{r+1},\frac{10r+2}{r+1}\right)$

$(\overrightarrow{OQ}\cdot\overrightarrow{OA})-\frac{1}{5}|\overrightarrow{OP}\times\overrightarrow{OA}|^2=10$

$\overrightarrow{OQ}=5\hat{i}+5\hat{j}+10\hat{k}$

$\overrightarrow{OA}=\frac{5r-1}{r+1}\hat{i}+\frac{5r-1}{r+1}\hat{j}+\frac{10r+2}{r+1}\hat{k}$

$\overrightarrow{OP}=-\hat{i}-\hat{j}+2\hat{k}$

$\overrightarrow{OP}\times\overrightarrow{OA}=\frac{1}{r+1}\left|\ \frac{5r-1}{5}\quad 5r-1\quad 10r+2\right.$

$=\frac{1}{r+1}(\hat{i}(20r)-\hat{j}(20r))$

$=5\left(\frac{5r-1}{r+1}\right)+5\left(\frac{5r-1}{r+1}\right)+10\left(\frac{10r+2}{r+1}\right)$

$-\frac{1}{5}\left(\frac{2\times400r^2}{(r+1)^2}\right)=10$

$\frac{150r+10}{r+1}-\frac{1}{5}\left(\frac{2\times400r^2}{(r+1)^2}\right)=10$

$(150r+10)(r+1)-160r^2=10(r+1)^2$

$(15r+1)(r+1)-16r^2=(r+1)^2$

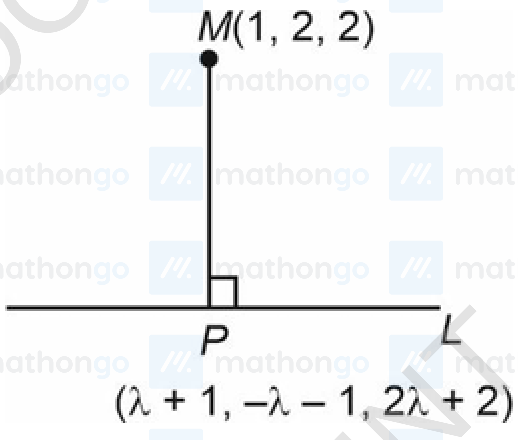
$15r^2+16r+1-16r^2=r^2+2r+1$

$-2r^2+14r=0$

$r=0,7$

Q19. General point on line L : $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z-2}{2}$

(4) is $(\lambda+1,-\lambda-1,2\lambda+2)$



DR's of PM are $(\lambda,-\lambda-3,2\lambda)$

$PM\perp L$

$\Rightarrow\lambda+(-1)(-\lambda-3)+2(2\lambda)=0$

$\Rightarrow 6\lambda+3=0$ $\Rightarrow$ $\boxed{\lambda=\frac{-1}{2}}$

mathongo FOCUS POINT

$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$

Let another line $L' : \frac{x+1}{1} = \frac{y-1}{-1} = \frac{z+2}{1}$

General point on line L' is $(\mu - 1, -\mu + 1, \mu - 2)$

Point of intersection of line L and L' is

$\lambda + 1 = \mu - 1$	$2\lambda + 2 = \mu - 2$
$\Rightarrow \mu - \lambda = 2 \ldots (1)$	$\Rightarrow 2\lambda = \mu - 4$

$\Rightarrow \boxed{\lambda = -2}$ and $\boxed{\mu = 0}$

$Q(-1, 1, -2)$

$$2(PQ)^2 = 2\left(\left(\frac{1}{2} + 1\right)^2 + \left(\frac{-1}{2} - 1\right)^2 + (1+2)^2\right)$$

$$= 2\left(\frac{9}{4} + \frac{9}{4} + 9\right)$$

$$= 27$$

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