CHAPTER 1 Units and Measurements

Systems of Units

1. The angle of 1' (minute of arc) in radian is nearly equal to, *(2020-Covid)*

a. 4.85×10^{-4} rad b. 4.80×10^{-6} rad
c. 1.75×10^{-2} rad d. 2.91×10^{-4} rad

2. The unit of thermal conductivity is *(2019)*

a. $W\ m^{-1}K^{-1}$ b. $J\ m\ K^{-1}$ c. $J\ m^{-1}K^{-1}$ d. $W\ m\ K^{-1}$

Dimensions of Physical Quantities

3. The quantities which have the same dimensions as those of solid angle are: *(2024)*

a. strain and arc b. angular speed and stress
c. strain and angle d. stress and angle

4. A force defined by $F = \alpha t^2 + \beta t$ acts on a particle at given time t. The factor which is dimensionless, if α and β are constants, is: *(2024)*

a. $\alpha\beta t$ b. $\alpha\beta/t$ c. $\beta t/\alpha$ d. $\alpha t/\beta$

5. Plane angle and solid angle have: *(2022)*

a. Both units and dimension
b. Units but no dimensions
c. Dimensions but no units
d. No units and no dimensions

6. The dimension $[MLT^{-2}A^{-2}]$ belong to the: *(2022)*

a. electric permittivity b. magnetic flux
c. self inductance d. magnetic permeability

7. If E and G respectively denote energy and gravitational constant, then $\frac{E}{G}$ has the dimensions of: *(2021)*

a. $[M][L^{-1}][T^{-1}]$ b. $[M][L^{0}][T^{0}]$
c. $[M^2][L^{-2}][T^{-1}]$ d. $[M^2][L^{-1}][T^{0}]$

8. Dimensions of stress are: *(2020)*

a. $[ML^2T^{-2}]$ b. $[ML^0T^{-2}]$ c. $[ML^{-1}T^{-2}]$ d. $[MLT^{-2}]$

Application of Dimensions

9. If force [F], acceleration [A] and time [T] are chosen as the fundamental physical quantities. Find the dimensions of energy. *(2021)*

a. $[F][A][T^2]$ b. $[F][A][T^{-1}]$
c. $[F][A^{-1}][T]$ d. $[F][A][T]$

10. A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_0}$ is [c is velocity of light, G is universal constant of gravitation and e is charge]: *(2017-Delhi)*

a. $c^2\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$ b. $\frac{1}{c^2}\left[\frac{e^2}{G4\pi\varepsilon_0}\right]^{1/2}$

c. $\frac{1}{c^2}G\frac{e^2}{4\pi\varepsilon_0}$ d. $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$

11. Planck's constant (h), speed of light in vacuum (c) and Newton's gravitational constant (G) are three fundamental constants. Which of the following combinations of these has the dimension of length? *(2016-II)*

a. $\sqrt{\frac{hc}{G}}$ b. $\sqrt{\frac{Gc}{h^{3/2}}}$ c. $\frac{\sqrt{hG}}{c^{3/2}}$ d. $\sqrt{\frac{hG}{c^{5/2}}}$

12. If energy (E), velocity (V) and time (T) are chosen as the fundamental quantities, the dimensional formula of surface tension will be: *(2015)*

a. $[EV^{-1}T^{-2}]$ b. $[EV^{-2}T^{-2}]$ c. $[E^{-2}V^{-1}T^{-3}]$ d. $[EV^{-2}T^{-1}]$

13. If dimension of critical velocity of liquid flowing through a tube are expressed as $\nu_c \propto [\eta^x \rho^y r^z]$ where η, ρ and r are the coefficient of viscosity of liquid, density of liquid and radius of the tube respectively, then the values of x, y and z are given by: *(2015 - Re)*

a. 1, 1, 1 b. 1, –1, –1 c. –1, –1, 1 d. –1, –1, –1

14. If Force (F), Velocity (V) and Time (T) are taken as fundamental units, then the dimensions of mass are: *(2014)*

a. $[F\ V\ T^{-1}]$ b. $[F\ V\ T^{-2}]$ c. $[F\ V^{-1}\ T^{-1}]$ d. $[F\ V^{-1}T]$

Errors

15. The errors in the measurement which arise due to unpredictable fluctuations in temperature and voltage supply are: *(2023)*

a. Random errors b. Instrumental errors
c. Personal errors d. Least count errors

16. A metal wire has mass (0.4 ± 0.002) g, radius (0.3 ± 0.001) mm and length (5 ± 0.02) cm. The maximum possible percentage error in the measurement of density will nearly be : *(2023)*

a. 1.4% b. 1.2% c. 1.3% d. 1.6%

17. The percentage error in the measurement of g is:

(Given that $g = \frac{4\pi^2 L}{T^2}$, L (10 ± 0.1) cm, T = (100 ± 1) s) *(2022 Re)*

a. 7% b. 2% c. 5% d. 3%

18. The intervals measured by a clock given the following readings: 1.25 s, 1.24 s, 1.27 s, 1.21 s and 1.28 s. What is the percentage relative error in the observations? *(2020-Covid)*

a. 4% b. 16% c. 1.6% d. 2%

19. In an experiment, the percentage of error occurred in the measurement of physical quantities A, B, C and D are 1%, 2%, 3% and 4% respectively. Then the maximum percentage of error in the measurement of X, where $X = \frac{A^2 B^{1/2}}{C^{1/3} D^3}$ will be *(2019)*

a $\left(\frac{3}{13}\right)\%$ b. 16% c. – 10% d. 10%

20. In an experiment four quantities a, b, c and d are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity P is calculated as follows $P = \frac{a^3 b^2}{cd}$. % error in P is: *(2013)*

a. 4% b. 14% c. 10% d. 7%

Significant Figures

21. The diameter of a spherical bob, when measured with vernier callipers yielded the following values: 3.33 cm, 3.32 cm, 3.34 cm, 3.33 cm and 3.32 cm. The mean diameter to appropriate significant figures is: *(2023-Manipur)*

a. 3.328 cm b. 3.3 cm c. 3.33 cm d. 3.32 cm

22. The area of a rectangular field (in m^2) of length 55.3 m and breadth 25 m after rounding off the value for correct significant digits is : *(2022)*

a. 14×10^2 b. 138×10^1
c. 1382 d. 1382.5

23. Taking into account of the significant figures, what is the value of 9.99 m – 0.0099 m? *(2020)*

a. 9.98 m b. 9.980 m c. 9.9 m d. 9.9801 m

Measuring Instruments

24. The pitch of an error free screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a thick wire, the pitch scale reads 1 mm and 63rd division on the circular scale coincides with the reference line. The diameter of the wire is: *(2024 Re)*

a. 1.63 cm b. 0.163 cm c. 0.163 m d. 1.63 m

25. In a vernier calipers, $(N + 1)$ divisions of vernier scale coincide with N divisions of main scale. If 1 MSD represents 0.1 mm, the vernier constant (in cm) is: *(2024)*

a. $100 N$ b. $10(N+1)$
c. $\frac{1}{10N}$ d. $\frac{1}{100(N+1)}$

26. A screw gauge gives the following readings when used to measure the diameter of a wire
Main scale reading: 0 mm
Circular scale reading: 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions on the circular scale. The diameter of the wire from the above data is: *(2021)*

a. 0.026 cm b. 0.26 cm c. 0.052 cm d. 0.52 cm

27. A screw gauge has least count of 0.01 mm and there are 50 divisions in its circular scale.
The pitch of the screw gauge is: *(2020)*

a. 0.25 mm b. 0.5 mm c. 1.0 mm d. 0.01 mm

28. A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 divisions above the reference level. If screw gauge has a zero error of –0.004 cm, the correct diameter of the ball is *(2018)*

a. 0.053 cm b. 0.525 cm c. 0.521 cm d. 0.529 cm

Answer Key

1. (d)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (d)
11. (c)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (b)
21. (c)	**22.** (a)	**23.** (a)	**24.** (b)	**25.** (d)	**26.** (c)	**27.** (b)	**28.** (d)		

Explanations

1. (d) 1 minute of arc $= 1' = \left(\frac{1}{60}\right)^0 = \frac{1}{60} \times \frac{\pi}{180}$ radian

$= 2.91 \times 10^{-4}$ radian

2. (a) $K = \frac{Qx}{A(T_1 - T_2)t}$, where Q is the amount of heat flow, x is the thickness of the slab, A is the area of cross-section, and t is the time taken.

$$K = \frac{J\,m}{m^2\,Ks} = \frac{J}{s} \cdot \frac{m}{m^2} \cdot \frac{1}{K} = Wm^{-1}K^{-1}$$

3. (c) Strain and angle are also dimensionless Solid angle is dimensionless.

4. (d) $F = \alpha t^2 + \beta t$

$[MLT^{-2}] = \alpha\,[T^2]$

$\alpha = [MLT^{-4}]$

$[MLT^{-2}] = \beta\,[T]$

$\beta = [MLT^{-3}]$

$$\frac{\alpha t}{\beta} = \frac{[MLT^{-4}][T]}{[MLT^{-3}]} = [M^0L^0T^0]$$

5. (b) Plane angle and solid angle are dimensionless physical quantities but have units.

6. (d) $[MLT^{-2}A^{-2}]$ is the dimensional formula for Magnetic permeability

7. (d) For energy, the dimensional formula is $[ML^2\,T^{-2}]$

For gravitational constant, the dimensional formula is $[M^{-1}\,L^3\,T^{-2}]$

$$\therefore \frac{[E]}{[G]} = \frac{[M^1L^2T^{-2}]}{[M^{-1}L^3T^{-2}]} = \left[M^{1+1}\,L^{2-3}\,T^{-2+2}\right]$$

$$\Rightarrow \left[\frac{E}{G}\right] = \left[M^2\,L^{-1}\,T^0\right]$$

8. (c) Stress $= \frac{\text{Force}}{\text{Area}}$

Dimension of stress $= \frac{[M^1L^1T^{-2}]}{[L^2]}$

Dimension of stress $= [M^1L^{-1}T^{-2}]$

9. (a) According to the question;

$E \propto F^aA^b\,T^c$

$\Rightarrow [M^1\,L^2\,T^2] \propto [M^1\,L^1\,T^{-2}]^a\,[LT^{-2}]^b\,[T]^c$

$\Rightarrow [M^1\,L^2\,T^{-2}] \propto [M^a]\,[L^{a+b}]\,[T^{-2a-2b+c}]$

Comparing both sides,

$\Rightarrow a = 1 \Rightarrow a + b = 2$

$\Rightarrow 1 + b = 2 \Rightarrow b = 2 - 1 = 1 \Rightarrow b = 1$

$\Rightarrow -2a - 2b + c = -2$

$\Rightarrow -2(1) - 2(1) + c = -2 \Rightarrow -2 - 2 + c = -2$

$\Rightarrow -4 + c = -2 \Rightarrow c = -2 + 4 \Rightarrow c = 2$

$E \propto [F]\,[A]\,[T^2]$

10. (d) $L = [c]^a\,[G]^b\left[\frac{e^2}{4\pi\varepsilon_0}\right]^c$

$= [LT^{-1}]^a\,[M^{-1}\,L^3\,T^{-2}]^b\,[ML^3\,T^{-2}]^c$

$= L^{a+3b+3c}\,T^{-a-2b-2c}\,M^{-b+c}$

$a + 3b + 3c = 1;\ -a - 2b - 2c = 0;\ -b + c = 0$

$b = \frac{1}{2}\quad c = \frac{1}{2}$ ↓

$a = -2$

$$L = c^{-2}G^{1/2}\left[\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$$

$$L = \frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$$

11. (c) $\ell \propto h^xG^yc^z$

$$M^0L^1T^0 = (ML^2T^{-1})^x\,(M^{-1}L^3T^{-2})^y\,(LT^{-1})^z$$

$$= M^{x-y}L^{2x+3y+z}T^{-x-2y-z}$$

Equating:

$$\left.\begin{aligned} x - y &= 0 \\ 2x + 3y + z &= 1 \\ -x - 2y - z &= 0 \end{aligned}\right\} \begin{aligned} &\Rightarrow x = \frac{1}{2}; y = \frac{1}{2}; z = -\frac{3}{2} \\ &\Rightarrow \ell \propto \frac{\sqrt{hG}}{c^{3/2}} \end{aligned}$$

12. (b) Surface tension (S.T.) $\propto [E]^a\,[V]^b\,[T]^c$

$[S.T.] \propto [ML^2T^{-2}]^a\,[LT^{-1}]^b\,[T]^c$

$[MT^{-2}] \propto M^a\,L^{2a+b}\,T^{-2a-b+c}$

On comparing both sides

$2a + b = 0,\ -2a - b + c = -2$

$a = 1,\ b = -2,\ c = -2$

we get

S.T. $= EV^{-2}\,T^{-2}$

13. (b) $v_c \propto [\eta^x\rho^yr^z]$

$$[L^1T^{-1}] \propto [M^1L^{-1}T^{-1}]^x\,[M^1L^{-3}]^y\,[L^1]^z$$

$$[L^1T^{-1}] \propto [M^{x+y}][L^{-x-3y+z}][T^{-x}]$$

Taking comparison on both size

$x + y = 0,\ -x - 3y + z = 1,\ -x = -1$

$x = 1,\ y = -1,\ z = -1$

14. (d) $[\text{Mass}] = \left[\frac{\text{Force}}{\text{Acceleration}}\right] = \left[\frac{\text{Force}}{\text{Velocity/time}}\right]$

$= [FV^{-1}T]$

15. (a) As the factors controlling temperature and voltage supply are beyond prediction and control so the error occurred due to unpredictable fluctuations of temperature and voltage would be random errors.

16. (d) $m = \rho\pi r^2 l$

$$\rho = \frac{m}{\pi r^2 l}\quad \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta l}{l}$$

$$\frac{\Delta\rho}{\rho}\times 100 = \frac{0.002}{0.4}\times 100 + \frac{2\times 0.001}{0.3}\times 100 + \frac{0.02}{5}\times 100$$

$= \frac{0.2}{0.4} + \frac{0.2}{0.3} + \frac{2}{5}$

$= 0.5 + 0.67 + 0.4$

$= 1.57 = 1.6\%$

17. (d) For finding the percentage error in the measurement of g,

$$g = \frac{4\pi^2L}{T^2}$$

$$\frac{\Delta g}{g}\times 100 = \frac{\Delta L}{L}\times 100 + 2\frac{\Delta T}{T}\times 100$$

$$= \left(\frac{0.1}{10}\times 100\right) + 2\left[\frac{1}{100}\times 100\right]$$

$$\frac{\Delta g}{g}\times 100 = 1 + 2 = 3\%$$

18. (c) Mean of given observations

$$= \frac{1.25 + 1.24 + 1.27 + 1.21 + 1.28}{5} = 1.25 \text{ sec}$$

Mean of errors

$$= \frac{0 + 0.01 + 0.02 + 0.04 + 0.03}{5} = \frac{0.1}{5}$$

$$\%\ \text{Error} = \frac{0.1\times 100}{5\times 1.25} = 1.6\%$$

19. (b) $X = \frac{A^2B^{\frac{1}{2}}}{C^{\frac{1}{3}}D^3}$

$$\%\text{error}, \frac{\Delta X}{X}\times 100 = 2\frac{\Delta A}{A}\times 100 + \frac{1}{2}\frac{\Delta B}{B}\times 100 + \frac{1}{3}\frac{\Delta C}{C}\times 100 + 3\frac{\Delta D}{D}\times 100$$

$= 2\% + 1\% + 1\% + 12\%$

$= 16\%$

20. (b) $P = \frac{a^3b^2}{cd} \Rightarrow \frac{\Delta P}{P} = \pm\left(3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d}\right)$

$= \pm(3\times 1 + 2\times 2 + 3 + 4) \Rightarrow \pm 14\%$

21. (c) We have given the diameter of the spherical bob when measured with vernier callipers, now

$$\text{Mean diameter} = \frac{d_1 + d_2 + d_3 + d_4 + d_5}{5}$$

$$= \frac{3.33 + 3.32 + 3.34 + 3.33 + 3.32}{5}$$

$= 3.328 \approx 3.33$

22. (a) Area = Length × Breadth

$= 55.3 \times 25 = 1382.5 = 14 \times 10^2$

Area should have 2 significant figures, as the minimum significant figure in the data given is two.

23. (a) In subtraction the number of decimal places int he result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places.

$$\begin{array}{r} 9.99 \\ -0.0099 \\ \hline 9.98 \end{array} \rightarrow \text{3 significant figures.}$$

24. (b) Least count

$= \frac{\text{Pitch}}{\text{Divisions}} = \frac{1\text{ mm}}{100} = 0.01\text{ mm}$.

Reading:

MSR = 1 mm

CSR = 63 × 0.01 = 0.63

Total reading = 1.63 mm = 0.163 cm.

25. (d) (N + 1) vernier scale division (VSD) = (N) main scale division

$$1\,\text{VSD} = \left(\frac{N}{N+1}\right)\text{MSD}$$

L.C = Least count = 1 MSD – 1 VSD

$$= \left(1 - \frac{N}{N+1}\right)\text{MSD}$$

$$= \left(\frac{1}{N+1}\right)\text{MSD}$$

$$= \left(\frac{1}{N+1}\right) \times \frac{0.1}{10}\ \text{cm}$$

$$= \frac{1}{100(N+1)}\ \text{cm}$$

26. (c) Pitch of the screw gauge, P = 1 mm

Number of circular division, n = 100

Least count, L.C. = $\frac{P}{n} = \frac{1}{100} = 0.01$ mm

L.C. = 0.001 cm

Observation

MSR = 0 mm & CSR = 52 divisions

So, diameter of the wire,

D = MSR + (CSR × LC)

D = 0 + (52 × 0.001)

D = 0.052 cm

27. (b)

$$\text{Least count} = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

$$\Rightarrow 0.01\text{mm} = \frac{\text{Pitch}}{50}$$

⇒ Pitch = 0.5 mm.

28. (d) Reading = MSR + (n × LC) + Zero error

= 0.5 + (25 × 0.001) + 0.004

= 0.529 cm

CHAPTER 2

Motion in a Straight Line

Distance, Displacement, Speed and Velocity

1. A particle is moving along x-axis with its position (x) varying with time (t) as $x = \alpha t^4 + \beta t^2 + \gamma t + \delta$. The ratio of its initial velocity to its initial acceleration, respectively, is: *(2024 Re)*

a. $2\alpha : \delta$ b. $\gamma : 2\delta$ c. $4\alpha : \beta$ d. $\gamma : 2\beta$

2. A vehicle travels half the distance with speed v and the remaining distance with speed 2v. Its average speed is: *(2023)*

a. $\frac{3v}{4}$ b. $\frac{v}{3}$ c. $\frac{2v}{3}$ d. $\frac{4v}{3}$

3. The position of a particle is given by

$$\vec{r}(t) = 4t\hat{i} + 2t^2\hat{j} - 5\hat{k}$$

where t is in seconds and r in metre. Find the magnitude and direction of velocity $v(t)$, at $t = 1$ s, with respect to x-axis *(2023-Manipur)*

a. $4\sqrt{2}\ \text{ms}^{-1}, 45°$ b. $4\sqrt{2}\ \text{ms}^{-1}, 60°$

c. $3\sqrt{2}\ \text{ms}^{-1}, 30°$ d. $3\sqrt{2}\ \text{ms}^{-1}, 45°$

4. If $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{r} = 3\hat{i} + 2\hat{j} - 2\hat{k}$, then the scalar and vector products of $\vec{F}$ and $\vec{r}$ have the magnitudes respectively as: *(2022 Re)*

a. 10, 2 b. $3\sqrt{3}$ c. $4\sqrt{5}$ d. $10\sqrt{2}$

5. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be: *(2017-Delhi)*

a. $\frac{t_1 t_2}{t_2 - t_1}$ b. $\frac{t_1 t_2}{t_2 + t_1}$ c. $t_2 - t_1$ d. $\frac{t_1 + t_2}{2}$

6. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $X_P(t) = at + bt^2$ and $X_Q(t) = ft - t^2$. At what time do the cars have the same velocity? *(2016 - II)*

a. $\frac{a+f}{2(1+b)}$ b. $\frac{f-a}{2(1+b)}$ c. $\frac{a-f}{1+b}$ d. $\frac{a+f}{2(b-1)}$

7. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1 s and 2 s is: *(2016 - I)*

a. $\frac{3}{2}A + 4B$ b. $3A + 7B$

c. $\frac{3}{2}A + \frac{7}{3}B$ d. $\frac{A}{2} + \frac{B}{3}$

Acceleration

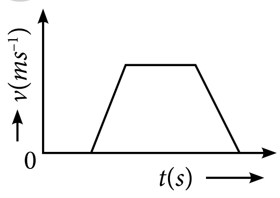
8. A particle of unit mass undergoes one dimensional motion such that its velocity varies according to $v(x) = \beta x^{-2n}$ where β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x, is given by: *(2015)*

a. $-2n\beta^2 x^{-4n-1}$ b. $-2\beta^2 x^{-2n+1}$

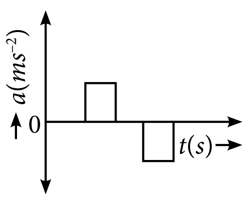
c. $-2n\beta^2 e^{-4n+1}$ d. $-2n\beta^2 x^{-2n-1}$

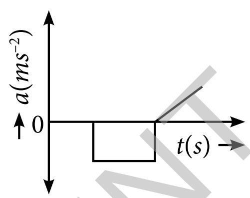
Graphs

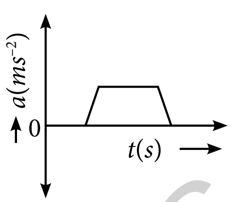
9. The velocity (v) – time (t) plot of the motion of a body is shown below: *(2024)*

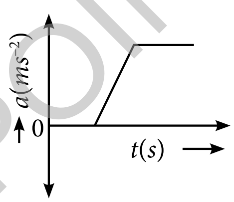


The acceleration (a) – time (t) graph that best suits this motion is:

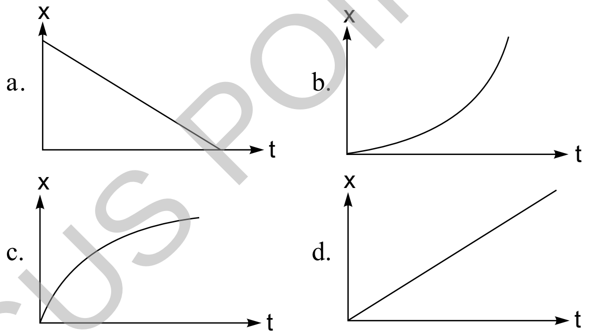
a. 

b. 

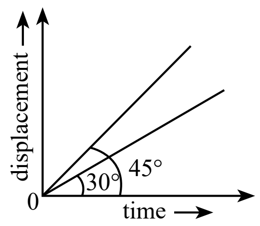
c. 

d. 

10. The position-time ($x - t$) graph for positive acceleration is: *(2022 Re)*



11. The displacement time graphs of two moving particle make angles of 30° and 45° with the x-axis as shown in the figure. The ratio of their respective velocity is : *(2022)*



a. $1:\sqrt{3}$ b. $\sqrt{3}:1$ c. 1 : 1 d. 1 : 2

Motion under Gravity

12. The ratio of the distance traveled by a freely falling body in the 1st, 2nd, 3rd and 4th second : *(2022)*

a. 1 : 1 : 1 : 1 b. 1 : 2 : 3 : 4
c. 1 : 4 : 9 : 16 d. 1 : 3 : 5 : 7

13. A ball is thrown vertically downward with a velocity of 20 m/s from the top of a tower. It hits the ground after some time with a velocity of 80m/s. The height of the tower is : ($g = 10$ m/s^2) *(2020)*

a. 340 m b. 320 m c. 300 m d. 360 m

14. A person sitting in the ground floor of a building notices through the window, of height 1.5 m, a ball dropped from the roof of the building crosses the window in 0.1 s. What is the velocity of the ball when it is at the **topmost point** of the window? (g = 10 m/s^2) *(2020-Covid)*

a. 14.5 m/s b. 4.5 m/s c. 20 m/s d. 15.5 m/s

15. A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between h_1, h_2 and h_3 is: *(2013)*

a. $h_1 = h_2 = h_3$ b. $h_1 = 2h_2 = 3h_3$
c. $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$ d. $h_2 = 3h_1$ and $h_3 = 3h_2$

Answer Key

1. (d) **2.** (d) **3.** (a) **4.** (d) **5.** (b) **6.** (b) **7.** (c) **8.** (a) **9.** (a) **10.** (b)
11. (a) **12.** (d) **13.** (c) **14.** (a) **15.** (c)

Explanations

1. (d) Given position:
$x = \alpha t^4 + \beta t^2 + \gamma t + \delta$
Velocity:
$$v = \frac{dx}{dt} = 4\alpha t^3 + 2\beta t + \gamma$$
At $t = 0$: $v = \gamma$
Acceleration:
$$a = \frac{dv}{dt} = 12\alpha t^2 + 2\beta$$
At $t = 0$: a = 2β
Ratio: $\frac{v}{a} = \frac{\gamma}{2\beta}$

2. (d) Average speed $= \left(\frac{4v^2}{3v}\right)$
$$= \frac{4v}{3}$$

3. (a) Velocity of the particle,
$$\vec{V} = \frac{d\vec{r}}{dt} = 4\hat{i} + 4t\hat{j} + 0\hat{k}$$
at $t = 1$ sec
$$\vec{V} = 4\hat{i} + 4(1)\hat{j}$$
$$|\vec{V}| = \sqrt{4^2 + 4^2} = 4\sqrt{2}$$
$$\tan\alpha = \frac{4}{4} = 1$$
$\alpha = 45°$

4. (d) Scalar product = $\vec{F}\cdot\vec{r}$
$= 2 \times 3 + 1 \times 2 + (-1) \times (-2)$
$= 6 + 2 + 2 = 10$

Vector product = $\vec{F} \times \vec{r}$
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 3 & 2 & -2 \end{vmatrix}$$
$= \hat{i}(-2 + 2) - \hat{j}(-4 + 3) + \hat{k}(4 - 3)$
$$\vec{F} \times \vec{r} = \hat{j} + \hat{k}$$
$$|\vec{F} \times \vec{r}| = \sqrt{2}$$

5. (b) t_1 = time taken at stationary escalator
t_2 = time taken at moving escalator
t = time taken in walking up the moving escalator
d = displacement of escalator
V_1 = velocity of Preeti and V_2 = velocity of escalator
$$(V_1 + V_2) = \frac{d}{t}$$
$$V_1 = \frac{d}{t}, V_2 = \frac{d}{t_2}$$
$$\frac{d}{t_1} + \frac{d}{t_2} = \frac{d}{t}$$
$$t = \frac{t_1\,t_2}{t_2 + t_2}$$

6. (b) $X_p(t) = at + bt^2$ $V_p = \frac{dX_p}{dt} = a + 2bt$

$X_Q(t) = ft - t^2$ $V_Q = \frac{dX_Q}{dt} = f - 2t$

as $V_p = V_Q$
$a + 2bt = f - 2t \Rightarrow t = \frac{f - a}{2(1 + b)}$

7. (c) We know, differentiation of displacement with respect to time gives velocity. v = At + Bt²

$$\frac{dx}{dt} = At + Bt^2$$

$$\int_0^x dx = \int_1^2 \left(At + Bt^2\right)dt$$

$$x = \frac{A}{2}\left(2^2 - 1^2\right) + \frac{B}{3}\left(2^3 - 1^3\right) = \frac{3A}{2} + \frac{7B}{3}$$

8. (a)

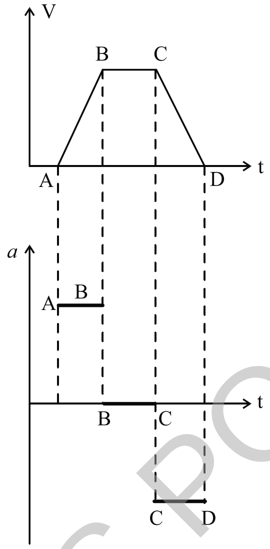
Velocity $= \beta x^{-2n}$

Differentiate velocity with respect to x, $\frac{dv}{dx} = -2n\beta x^{-2n-1}$

Now Acceleration $a = v\frac{dv}{dx} = \left(\beta x^{-2n}\right)\left(-2n\beta x^{-2n-1}\right)$

$\Rightarrow a = -2n\beta^2 x^{-4n-1}$

9. (a)



a = slope of $v - t$ graph

from A to B → slope is positive and constant

→ so acceleration is positive & constant

B to C →slope is zero ⇒ acceleration is zero.

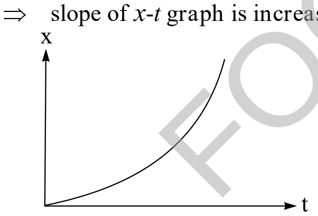
C to D → slope is negative and constant

→ so acceleration is negative & constant

10. (b) Positive acceleration shows acceleration in the direction of velocity,

$\frac{dv}{dt} > 0$ so, velocity is increasing

⇒ slope of x-t graph is increasing



11. (a) ∵ Velocity is equal to slope of x-t graph

⇒ v ∝ tan θ

$$\frac{v_1}{v_2} = \frac{\tan 30°}{\tan 45°} = \frac{1}{\sqrt{3}}$$

12. (d) distance travelled during nth second is given by

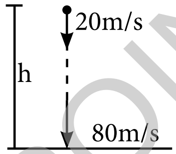
$$S_{nth} = u + \frac{a}{2}(2n-1) = 0 + \frac{a}{2}(2n-1)$$

[As the body is starting from rest, ∴ u = 0]

$S_{nth} = (2n - 1) \Rightarrow S_{1st} : S_{2nd} : S_{3rd} : S_{4th}$

= [2(1) – 1] : [2(2) – 1] : [2(3) – 1] : [2(4) – 1] = 1 : 3 : 5 : 7

13. (c)



v is the velocity of the ball before it strikes the earth and u in the initial velocity

$v^2 = u^2 + 2gh$

$80^2 = 20^2 + 2 \times 10h$

h = 300m

14. (a) From equation of motion

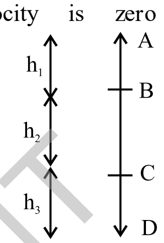
$$S = ut + \frac{1}{2}at^2$$

here S = 1.5 m and t = 0.1s

$$1.5 = u(0.1) + \frac{1}{2}(10)(0.1)(0.1)$$

⇒ u = 14.5 m/s

15. (c) Here initial velocity is zero



$AB = h_1 = \frac{1}{2}g(5)^2$

$\Rightarrow h_1 = 125$ m (∵ u = 0)

$h_2 = BC = \frac{1}{2}g\,[10^2 - 5^2]$

$\Rightarrow h_2 = 375$ m

$h_3 = CD = \frac{1}{2}g\left[15^2 - 10^2\right]$

$h_3 = 625$ m

$h_1 : h_2 : h_3$

125 : 375 : 625 = 1 : 3 : 5

$\Rightarrow h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

CHAPTER 3

Motion in a Plane

Vectors

1. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is: *(2016 - I)*

a. 1° b. 90° c. 45° d. 180°

2. A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\hat{x} + \sin\omega t\hat{y}$ where ω is a constant Which of the following is true? *(2016 - I)*

a. Velocity and acceleration both are perpendicular to $\vec{r}$

b. Velocity and acceleration both are parallel to $\vec{r}$

c. Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin

d. Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin

3. If vectors $\vec{A} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$ and $\vec{B} = \cos\frac{\omega t}{2}\hat{i} + \sin\frac{\omega t}{2}\hat{j}$ are functions of time, then the value of t at which they are orthogonal to each other are: *(2015 Re)*

a. $t = 0$ b. $t = \frac{\pi}{4\omega}$

c. $t = \frac{\pi}{2\omega}$ d. $t = \frac{\pi}{\omega}$

Motion in a Plane with Constant Acceleration

4. The two-dimensional motion of a particle, described by $\vec{r} = (\hat{i} + 2\hat{j})A\cos\omega t$ is a/an:

A. parabolic path B. elliptical path

C. periodic motion D. simple harmonic motion

Choose the correct answer from the options given below: *(2024 Re)*

a. B, C and D only b. A, B and C only

c. A, C and D only d. C and D only

5. A football player is moving southward and suddenly turns eastward with the same speed to avoid an opponent. The force that acts on the player while turning is: *(2023)*

a. along south-west b. along eastward

c. along northward d. along north-east

6. The '*x*' and '*y*' coordinates of the particle at any time are '*x*' = $5t - 2t^2$ and '*y*' = 10t, respectively, where '*x*' and '*y*' are in metres and 't' in seconds. The acceleration of the particle at **t = 2** s is: *(2017-Delhi)*

a. 5 m/s² b. –4 m/s² c. –8 m/s² d. 0

7. The position vector of a particle $\vec{R}$ as a function of time is given by:

$$\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$$

Where R is in metres, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x and y-direction, respectively. Which one of the following statements is wrong for the motion of particle? *(2015)*

a. Path of the particle is a circle of radius 4 metre

b. Acceleration vectors is along $-\vec{R}$

c. Magnitude of acceleration vector is $\frac{v^2}{R}$ where v is the velocity of particle.

d. Magnitude of the velocity of particle is 8 metre/second

8. A particle is moving such that its position coordinates (x, y) are:

(2 m, 3 m) at time t = 0,

(6 m, 7 m) at time t = 2 s and

(13 m, 14 m) at time t = 5 s

Average velocity vector $(\vec{v}_{av})$ from t = 0 to t = 5 s is: *(2014)*

a. $\frac{1}{5}(13\hat{i} + 14\hat{j})$ b. $\frac{7}{3}(\hat{i} + \hat{j})$ c. $2(\hat{i} + \hat{j})$ d. $\frac{11}{5}(\hat{i} + \hat{j})$

Projectile Motion

9. A bullet is fired from a gun at the speed of 280 m/s in the direction 30º above the horizontal. The maximum height attained by the bullet is *(2023)*

($g = 9.8\ ms^{-2}$, sin 30º = 0.5)

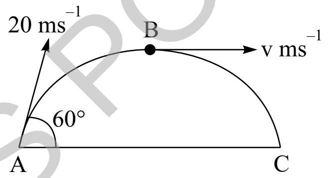
a. 3000 m b. 2800 m c. 2000 m d. 1000 m

10. A horizontal bridge is built across a river. A student standing on the bridge throws a small ball vertically upwards with a velocity 4 ms^{-1}. The ball strikes the water surface after 4 s. The height of bridge above water surface is

(Take $g = 10\ ms^{-2}$): *(2023)*

a. 68 m b. 56 m c. 60 m d. 64 m

11. A ball is projected from point A with velocity 20 ms^{-1} at an angle 60° to the horizontal direction. At the highest point B of the path (as shown in figure), the velocity v ms^{-1} of the ball will be: *(2023-Manipur)*



a. 20 b. $10\sqrt{3}$ c. Zero d. 10

12. A cricket ball is thrown by a player at a speed of 20 m/s in a direction 30° above the horizontal. The maximum height attained by the ball during its motion is: *(2022 Re)*
(g = 10 m/s²)

a. 25 m b. 5 m c. 10 m d. 20 m

13. A ball is projected with a velocity, 10 ms^{-1}, at an angle of 60° with the **vertical direction**. Its speed at the highest point of its trajectory will be: *(2022)*

a. 10 ms^{-1} b. Zero c. $5\sqrt{3}\ ms^{-1}$ d. 5 ms^{-1}

14. A particle moving in a circle of radius R with a uniform speed takes a time T to complete one revolution.
If this particle were projected with the same speed at an angle 'θ' to the horizontal, the maximum height attained by it equals 4R. The angle of projection, θ, is then given by: *(2021)*

a. $\theta = \cos^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{1/2}$ b. $\theta = \sin^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{1/2}$

c. $\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{1/2}$ d. $\theta = \cos^{-1}\left(\frac{gT^2}{\pi^2 R}\right)^{1/2}$

15. A car starts from rest and accelerates at 5 m/s². At t = 4 s, a ball is dropped out of a window by a person sitting in the car. What is the velocity and acceleration of the ball at t = 6s ? *(2021)*

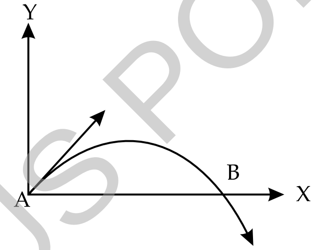
a. 20 m/s, 0 b. $20\sqrt{2}$ m/s, 0

c. $20\sqrt{2}$ m/s, 10 m/s² d. 20 m/s, 5 m/s²

16. A projectile is fired from the surface of the earth with a velocity of 5 ms^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 ms^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is: (given g = 9.8 ms^{-2}) *(2014)*

a. 3.5 b. 5.9 c. 16.3 d. 110.8

17. The velocity of a projectile at the initial point A $(2\hat{i} + 3\hat{j})$ m/s. Its velocity (in m/s) at point B is: *(2013)*



a. $2\hat{i} + 3\hat{j}$ b. $-2\hat{i} - 3\hat{j}$

c. $-2\hat{i} + 3\hat{j}$ d. $2\hat{i} - 3\hat{j}$

Relative Velocity in 2-D

18. The speed of a swimmer in still water is 20 m/s. The speed of river water is 10 m/s and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path, the angle at which he should make his strokes w.r.t. north is given by: *(2019)*

a. 30° west b. 0° c. 60° west d. 45° west

19. A ship A is moving Westwards with a speed of 10 km/h and a ship B 100 km South of A, is moving Northwards with a speed of 10 km/h. The time after which the distance between them becomes shortest, is: *(2015)*

a. 5 h b. $5\sqrt{2}$ h

c. $10\sqrt{2}$ h d. 0 h

Uniform Circular Motion

20. Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. The ratio of angular speed of A to that of B will be: *(2019)*

a. $r_A : r_B$ b. $v_A : v_B$ c. $r_B : r_A$ d. 1 : 1

21. When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel x_2 distance. Then $x_1 : x_2$ will be: *(2019)*

a. $1:\sqrt{2}$ b. $\sqrt{2}:1$

c. $1:\sqrt{3}$ d. $1:2\sqrt{3}$

Answer Key

1. (b)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (d)	**9.** (d)	**10.** (d)
11. (d)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (c)	**16.** (a)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (d)
21. (c)									

Explanations

1. (b) Let the two vectors be $\vec{A}$ and $\vec{B}$ with magnitudes A and B respectively.

$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$4AB\cos\theta = 0$

$\cos\theta = 0$

$\theta = 90°$

2. (c) $\vec{r} = \cos\omega t\hat{x} + \sin\omega t\hat{y}$

$\hat{v} = -(\omega\sin\omega t)\hat{x} + (\omega\cos\omega t)\hat{y}$

$\vec{a} = -(\omega^2\cos\omega t)\hat{x} + \omega^2(-\sin\omega t)\hat{y}$

$= -\omega^2\vec{r}$

$\vec{r}.\vec{v} = 0$ hence $\vec{r} \perp \vec{v}$

3. (d) Dot product of the two vectors $\vec{A}$ and $\vec{B}$ will be zero. because they are orthogonal to each other.

$\vec{A}.\vec{B} = 0$

$\left(\cos\omega t\cos\frac{\omega t}{2}\right) + \left(\sin\omega t\sin\frac{\omega t}{2}\right) = 0$

$\cos\left(\omega t - \frac{\omega t}{2}\right) = 0 \Rightarrow \cos\frac{\omega t}{2} = 0$

$\Rightarrow \frac{\omega t}{2} = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{\omega}$

4. (d) Let:

$x = A\cos\omega t$, $y = 2A\cos\omega t \Rightarrow y = 2x$

Straight-line motion along $y = 2x$, with both coordinates undergoing SHM.

Velocity

$\vec{v} = \frac{d\vec{r}}{dt} = -(\hat{i}+2\hat{j})A\omega\sin\omega t$

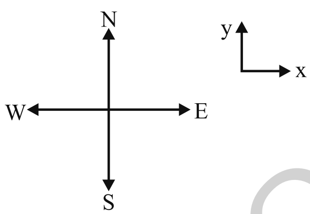
$a = \frac{d^2r}{dt^2}$

$= -(1+2\hat{j})\omega^2 A\cos\omega t$

Acceleration $\vec{a} = -\omega^2\vec{r}$, indicating Simple Harmonic Motion (SHM).

5. (d) Initial velocity $= -v\hat{j}$

Final velocity $= v\hat{i}$



Change in velocity $= v\hat{i} - (-v\hat{j})$

$= v(\hat{i}+\hat{j})$

Momentum gain is along $\hat{i}+\hat{j}$

$\Rightarrow$ Force experienced is along $\hat{i}+\hat{j}$

$\Rightarrow$ Force experienced is in North-East direction.

6. (b) $x = 5t - 2t^2$ and $y = 10t$

$v = \frac{dx}{dt} = 5 - 4t$, $v = \frac{dy}{dt} = 10$

$a_x = \frac{dv}{dt} = -4\ ms^{-2}$, $a_y = 0$

$a = -4 m/s^2$

7. (d) $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$

$\vec{v} = \frac{d\vec{R}}{dt} = 8\pi\cos 2\pi t\hat{i} - 8\pi\sin 2\pi t\hat{j}$

$|\vec{v}| = 8\ m/s$

$|\vec{v}| = \sqrt{[8\pi\cos(2\pi t)]^2 + [-8\pi\sin(2\pi t)]^2}$

$= \sqrt{64\pi^2}$

$= 8\pi$ m/s

$\therefore$ Statement in option (d) is wrong

8. (d) $\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(13-2)\hat{i} + (14-3)\hat{j}}{5-0} = \frac{11}{5}(\hat{i}+\hat{j})$

9. (d) $h_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{280\times 280}{2\times 9.8}\times\frac{1}{4}$

$= 1000$ m

10. (d) Let height of bridge = h

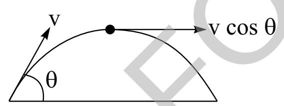
Displacement of ball, S = –h

$S = ut + \frac{1}{2}at^2$

$-h = 4\times 4 + \frac{1}{2}(-10)(4)^2$

$\Rightarrow h = 64$ m

11. (d)

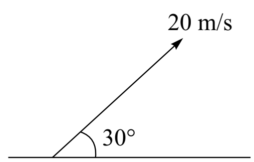


Velocity in horizontal direction remains same because there is no acceleration in horizontal direction

V at top = $v\cos\theta$

$= 20 \times \frac{1}{2} = 10$ m/s

12. (b)



The maximum height attained by the ball during its motion,

$H = \frac{u^2\sin^2\theta}{2g} = 5$ m

13. (c) At highest point only horizontal component of velocity remains

$\because$ angle with the vertical is 60°, therefore angle with horizontal is 30°

$\Rightarrow u_x = u\cos\theta = 10\cos 30°$

$= 5\sqrt{3}\ ms^{-1}$

14. (c) According to the question, the time period is given by,

$T = \frac{2\pi R}{V}$

$\Rightarrow V = \frac{2\pi R}{T}$ ...(i)

$H_{max} = \frac{V^2\sin^2\theta}{2g} = \frac{\left(\frac{2\pi R}{T}\right)^2\sin^2\theta}{2g}$

$\Rightarrow 4R = \frac{4\pi^2R^2\sin^2\theta}{2T^2g}$

$\Rightarrow 2T^2g = \pi^2R\sin^2\theta \Rightarrow \sin^2\theta = \frac{2T^2g}{\pi^2R}$

$\Rightarrow \sin^2\theta = \left(\frac{2gT^2}{\pi^2R}\right)^{\frac{1}{2}}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2R}\right)^{\frac{1}{2}}$

15. (c) Initial velocity, u = 0, Final velocity, v

acceleration, a = 5 m/s²

time, t = 4 sec

Now, using equation of motion,

$v = u + at \Rightarrow v = 0 + (5)(4)$

$\Rightarrow v = 20$ m/s

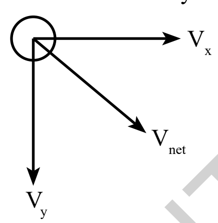
At t = 6 sec

For stone

acceleration due to gravity, a = g = 10 m/s²

As car is having, v = 20 m/s. So, stone also does. Horizontal velocity component, $V_x = 20$ m/s

Vertical downward velocity component, V_y



$V_y = u + at$

$V_y = 0 + g(6-4)$

$V_y = 10 \times 2 = 20$ m/s

$V_{net} = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{(20)^2 + (20)^2} = \sqrt{400+400} = \sqrt{800}$

$V_{net} = 20\sqrt{2}$ m/s

16. (a) If the trajectory is same for both the particles their, maximum height will be the same i.e.,

$(H_{max})_1 = (H_{max})_2$

$\Rightarrow \frac{u_1^2 \sin^2\theta}{2g_1} = \frac{u_2^2 \sin^2\theta}{2g_2}$

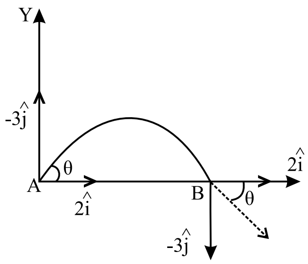
$\Rightarrow \frac{u_1^2}{u_2^2} = \frac{g_1}{g_2}$

$\Rightarrow g_2 = \frac{9.8\times 9}{25}$

$\Rightarrow g_2 = 3.5\ m/s^2$

17. (d) In a projectile vertical component of velocity keeps on changing with time.

While horizontal velocity component remains constant



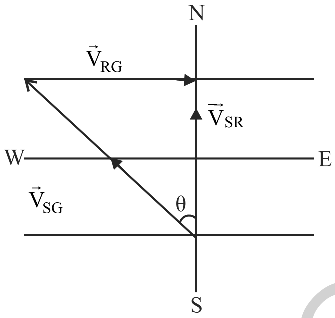
$\therefore$ Velocity is $2\hat{i} - 3\hat{j}$

18. (a) $V_{SG} = 20m/s$

$V_{RG} = 10m/s$

For shortest path

$\vec{V}_{SG} + \vec{V}_{RG} = \vec{V}_{SR}$



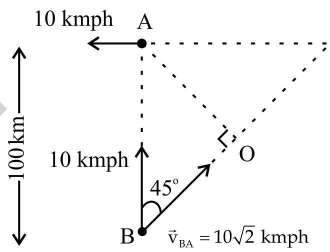
From vector triangle:-

$\sin\theta = \left|\frac{\vec{V}_{RG}}{\vec{V}_{SG}}\right|$

$\sin\theta = \frac{10}{20}$

$\sin\theta = \frac{1}{2} = \theta = 30°\text{west}$

19. (a)



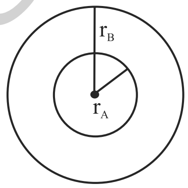
$|\vec{v}_{BA}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ kmph

Distance $OB = 100\cos 45° = 50\sqrt{2}$ km

Time taken to reach the shortest distance between

A & B $= \frac{50\sqrt{2}}{|\vec{v}_{BA}|} = \frac{50\sqrt{2}}{10\sqrt{2}} = 5$ h

20. (d)



$T_A = T_B = T$

$\omega_A = \frac{2\pi}{T_A} \Rightarrow \omega_B = \frac{2\pi}{T_B}$

$\frac{\omega_A}{\omega_B} = \frac{T_B}{T_A} = \frac{T}{T} = 1$

21. (c)

(Stopping distance) $x_1 = \frac{u^2}{2g\sin 60°}$

(Stopping distance) $x_2 = \frac{u^2}{2g\sin 30°}$

$\Rightarrow \frac{x_1}{x_2} = \frac{\sin 30°}{\sin 60°} = \frac{1\times 2}{2\times\sqrt{3}} = 1:\sqrt{3}$

CHAPTER 4

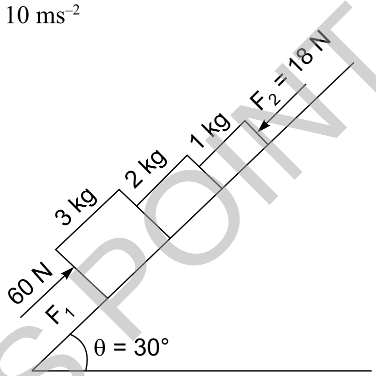
Laws of Motion

Equation of Motion and Newton's Laws of Motion

1. A box of mass 5 kg is pulled by a cord, up along a frictionless plane inclined at 30° with the horizontal. The tension in the cord is 30 N. The acceleration of the box is (Take $g = 10\ ms^{-2}$) *(2024 Re)*

a. $2\ m\ s^{-2}$ b. Zero c. $0.1\ m\ s^{-2}$ d. $1\ m\ s^{-2}$

2. In the diagram shown, the normal reaction force between 2 kg and 1 kg is (Consider the surface, to be smooth) : Given $g = 10\ ms^{-2}$ *(2022 Re)*

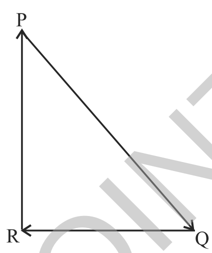


a. 10 N b. 25 N c. 39 N d. 6 N

3. A small block slides down on a smooth inclined plane, starting from rest at time t = 0. Let S_n be the distance travelled by the block in the interval t = n – 1 to t = n. The, the ratio $\frac{S_n}{S_{n+1}}$ is: *(2021)*

a. $\frac{2n-1}{2n+1}$ b. $\frac{2n+1}{2n-1}$ c. $\frac{2n}{2n-1}$ d. $\frac{2n-1}{2n}$

4. A particle moving with velocity $\overline{V}$ is acted by three forces shown by the vector triangle PQR. The velocity of the particle will : *(2019)*



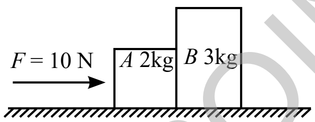
a. Increase
b. Decrease
c. Remain constant
d. Change according to the smallest force

5. A balloon with mass m is descending down with an acceleration a (where a < g). How much mass should be removed from it so that it starts moving up with an acceleration a? *(2014)*

a. $\frac{2ma}{g+a}$ b. $\frac{2ma}{g-a}$ c. $\frac{ma}{g+a}$ d. $\frac{ma}{g-a}$

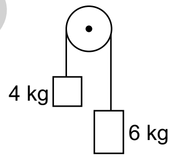
Motion of Connected Bodies

6. A horizontal force 10 N is applied to a block *A* as shown in figure. The mass of blocks *A* and *B* are 2 kg and 3 kg, respectively. The blocks slide over a frictionless surface. The force exerted by block *A* on block *B* is: *(2024)*



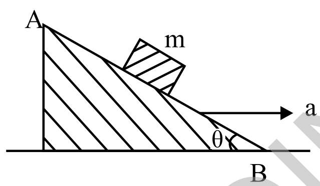
a. 6 N b. 10 N c. Zero d. 4 N

7. Two bodies of mass 4 kg and 6 kg are tied to the ends of a massless string. The string passes over a pulley which is frictionless (see figure). The acceleration of the system in terms of acceleration due to gravity (g) is: *(2020)*



a. g/2 b. g/5 c. g/10 d. g

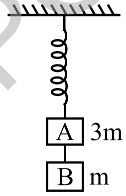
8. A block of mass m is placed on a smooth inclined wedge ABC of inclination θ as shown in the figure. The wedge is given an acceleration 'a' towards the right. The relation between a and θ for the block to remain stationary on the wedge is: *(2018)*



a. $a = g\cos\theta$ b. $a = \frac{g}{\sin\theta}$

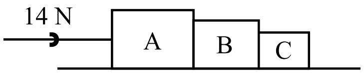
c. $a = \frac{g}{\text{cosec}\,\theta}$ d. $a = g\tan\theta$

9. Two blocks A and B of masses 3 m and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively: *(2017-Delhi)*



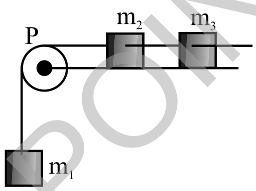
a. $g, \frac{g}{3}$ b. g, g   c. $\frac{g}{3}, \frac{g}{3}$ d. $\frac{g}{3}, g$

10. Three blocks A, B and C of masses 4 kg, 2 kg and 1 kg respectively, are in contact on a friction less surface, as shown. If a force of 14 N is applied on the 4 kg block, then the contact force between A and B is: *(2015)*



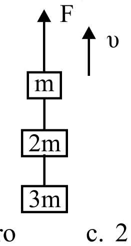
a. 6 N   b. 8 N   c. 18 N   d. 2 N

11. A system consists of three masses m_1, m_2 and m_3 is connected by a string passing over a pulley P. The mass m_1 hangs freely and m_2 and m_3 are on a rough horizontal table (the coefficient of friction = μ). The pulley is frictionless and of negligible mass. The downward acceleration of mass m_1 is: (Assume $m_1 = m_2 = m_3 = m$) *(2014)*



a. $\frac{g(1-g\mu)}{9}$ b. $\frac{2g\mu}{g}$ c. $\frac{g(1-2\mu)}{3}$ d. $\frac{g(1-2\mu)}{2}$

12. Three blocks with masses m, 2m and 3m are connected by strings, as shown in the figure. After an upward force F is applied on block m, the masses move upward at constant speed v. What is the net force on the block of mass 2m? (g is the acceleration due to gravity) *(2013)*



a. 6 mg   b. Zero   c. 2 mg   d. 3 mg

Conservation of Momentum and Impulse Momentum Theory

13. A bullet from a gun is fired on a rectangular wooden block with velocity u. When bullet travels 24 cm through the block along its length horizontally, velocity of bullet become $\frac{u}{3}$. Then it further penetrates into the block in the same direction before coming to rest exactly at the other end of the block. The total length of the block is: *(2023)*

a. 30 cm   b. 27 cm   c. 24 cm   d. 28 cm

14. A 1 kg object strikes a wall with velocity 1 ms^{-1} at an angle of 60° with the wall and reflects at the same angle. If it remains in contact with wall for 0.1 s, then the force exerted on the wall is: *(2023-Manipur)*

a. $30\sqrt{3}$ N   b. Zero   c. $10\sqrt{3}$ N   d. $20\sqrt{3}$ N

15. The distance covered by a body of mass 5 g having linear momentum 0.3 kg m/s in 5 s is: *(2022 Re)*

a. 0.3 m   b. 300 m   c. 30 m   d. 3 m

16. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R). *(2022 Re)*

Assertion (A): When a fire cracker (rocket) explodes in mid air, its fragments fly in such a way that they continue moving in the same path, which the fire cracker would have followed, had it not exploded.

Reason (R): Explosion of cracker (rocket) occurs due to internal forces only and no external force acts for this explosion.

In the light of the above statements, choose the most appropriate answer from the options given below:

a. (A) is not correct but (R) is correct
b. Both (A) and (R) are correct and (R) is the correct explanation of (A)
c. Both (A) and (R) are correct but (R) is not the correct explanation of (A)
d. (A) is correct but (R) is not correct.

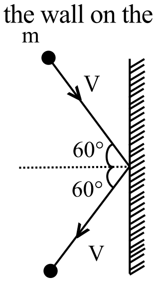
17. A shell of mass m is at **rest initially**. It explodes into three fragments having mass in the ratio 2 : 2 : 1. If the fragments having equal mass fly off along mutually perpendicular directions with speed v, the speed of the third (lighter) fragment is: *(2022)*

a. $3\sqrt{2}$ v   b. v   c. $\sqrt{2}$ v   d. $2\sqrt{2}$ v

18. A ball of mass 0.15 kg is dropped from a height 10 m, strikes the ground and rebounds to the same height. The magnitude of impulse imparted to the ball is (g = 10 m/s^2) nearly: *(2021)*

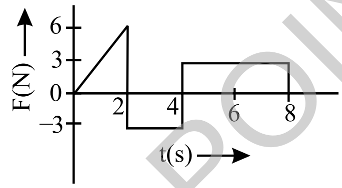
a. 4.2 kg m/s   b. 2.1 kg m/s
c. 1.4 kg m/s   d. 0 kg m/s

19. A rigid ball of mass m strikes a rigid wall at 60° and gets reflected without loss of speed as shown in the figure below. The value of impulse imparted by the wall on the ball will be: *(2016 - II)*



a. $\frac{mv}{2}$ b. $\frac{mv}{3}$ c. mv   d. 2mv

20. The force F acting on a particle of mass m is indicated by the force-time graph shown. The change in momentum of the particle over the time interval from zero to 8 s is: *(2014)*



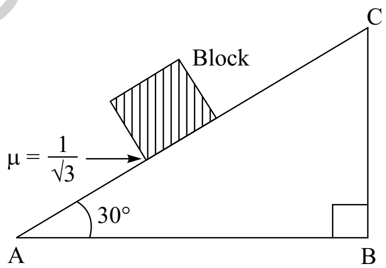
a. 24 Ns   b. 20 Ns   c. 12 Ns   d. 6 Ns

Friction

21. Calculate the maximum acceleration of a moving car so that a body lying on the floor of the car remains stationary. The coefficient of static friction between the body and the floor is 0.15 ($g = 10\ ms^{-2}$). *(2023)*

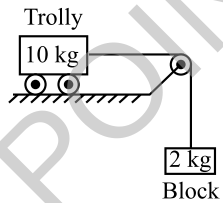
a. $50\ ms^{-2}$ b. $1.2\ ms^{-2}$ c. $150\ ms^{-2}$ d. $1.5\ ms^{-2}$

22. A block of mass 2 kg is placed on inclined rough surface AC (as shown in figure) of coefficient of friction μ. If $g = 10\ ms^{-2}$, the net force (in N) on the block will be: *(2023-Manipur)*



a. $10\sqrt{3}$ b. zero c. 10 d. 20

23. Calculate the acceleration of the block and trolly system shown in the figure. The coefficient of kinetic friction between the trolly and the surface is 0.05. ($g = 10\ m/s^2$, mass of the string is negligible and no other friction exists). *(2020-Covid)*



a. $1.50\ m/s^2$ b. $1.66\ m/s^2$ c. $1.00\ m/s^2$ d. $1.25\ m/s^2$

24. Which one of the following statements is incorrect? *(2018)*

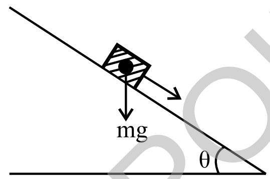
a. Frictionless force opposes the relative motion.
b. Limiting value of static friction is directly proportional to normal reaction.
c. Rolling friction is smaller than sliding friction.
d. Coefficient of sliding friction has dimensions of length.

25. A block A of mass m_1 rests on a horizontal table. A light string connected to it passes over a frictionless pulley at the edge of table and from its other end another block B of mass m_2 is suspended. The coefficient of kinetic friction between the block and table is μ_k. When the block A is sliding on the table, the tension in the string is: *(2015)*

a. $\frac{(m_2 - \mu_k m_1)g}{(m_1 + m_2)}$ b. $\frac{m_1 m_2 (1+\mu_k)g}{(m_1 + m_2)}$

c. $\frac{m_1 m_2 (1-\mu_k)g}{(m_1 + m_2)}$ d. $\frac{(m_2 + \mu_k m_1)g}{(m_1 + m_2)}$

26. A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4.0 m down the plank in 4.0 s. The coefficients of static and kinetic friction between the box and the plank will be, respectively: *(2015 - Re)*



a. 0.4 and 0.3 b. 0.6 and 0.6
c. 0.6 and 0.5 d. 0.5 and 0.6

27. The upper half of an inclined plane of inclination θ is perfectly smooth while lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom, if the coefficient of friction between the block and lower half of the plane is given by *(2013)*

a. $\mu = 2\tan\theta$ b. $\mu = \tan\theta$

c. $\mu = \frac{1}{\tan\theta}$ d. $\mu = \frac{2}{\tan\theta}$

Circular Motion, Banking of Road

28. A bob is whirled in a horizontal circle by means of a string at an initial speed of 10 rpm . If the tension in the string is quadrupled while keeping the radius constant, the new speed is: *(2024 Re)*

a. 20 rpm b. 40 rpm c. 5 rpm d. 10 rpm

29. A particle moving with uniform speed in a circular path maintains *(2024)*

a. constant velocity by varying acceleration.
b. varying velocity and varying acceleration.
c. constant velocity.
d. constant acceleration.

30. A bob is whirled in a horizontal plane by means of a string with an initial speed of ω rpm. The tension in the string is T. If speed becomes 20 ω while keeping the same radius, the tension in the string becomes: *(2024)*

a. $\frac{T}{4}$ b. $\sqrt{2}T$ c. T d. $4T$

31. A particle is executing uniform circular motion with velocity $\vec{v}$ and acceleration $\vec{a}$. Which of the following is true? *(2023-Manipur)*

a. $\vec{v}$ is a constant; $\vec{a}$ is not a constant
b. $\vec{v}$ is not a constant; $\vec{a}$ is not constant
c. $\vec{v}$ is a constant; $\vec{a}$ is a constant
d. $\vec{v}$ is not a constant; $\vec{a}$ is a constant

32. A block of mass 10 kg is in contact against the inner wall of a hollow cylindrical drum of radius 1 m. The coefficient of friction between the block and the inner wall of the cylinder is 0.1. The **minimum** angular velocity needed for the cylinder to keep the block stationary when the cylinder is vertical and rotating about its axis, will be : ($g = 10\ m/s^2$) *(2019)*

a. $\sqrt{10}$ rad / s b. $\frac{10}{2\pi}$ rad / s

c. 10 rad/s d. 10π rad/s

33. One end of string of length l is connected to a particle of mass 'm' and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed 'v', the net force on the particle (directed towards center) will be: (T represents the tension in the string) *(2017-Delhi)*

a. $T+\frac{mv^2}{l}$ b. $T-\frac{mv^2}{l}$ c. Zero d. T

34. A car is negotiating a curved road of radius R. The road is banked at an angle θ. The coefficient of friction between the tyres of the car and the road is μ_s. The maximum safe velocity on this road is: *(2016 - I)*

a. $\sqrt{gR^2\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$ b. $\sqrt{gR\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$

c. $\sqrt{\frac{g\mu_s+\tan\theta}{R\,1-\mu_s\tan\theta}}$ d. $\sqrt{\frac{g}{R^2}\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}}$

35. Two stones of masses m and 2 m are whirled in horizontal circles, the heavier one in a radius r/2 and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they experience same centripetal forces. The value of n is: *(2015 Re)*

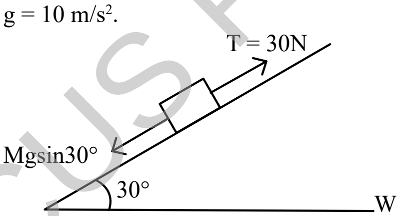
a. 1 b. 2 c. 3 d. 4

Answer Key

1. (d)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (d)	**10.** (a)
11. (c)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (a)	**19.** (c)	**20.** (c)
21. (d)	**22.** (b)	**23.** (d)	**24.** (d)	**25.** (b)	**26.** (c)	**27.** (a)	**28.** (a)	**29.** (b)	**30.** (d)
31. (b)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (b)					

Explanations

1. (d) Mass m = kg,
Inclination θ = 30°,
Tension T = 30 N,
g = 10 m/s².



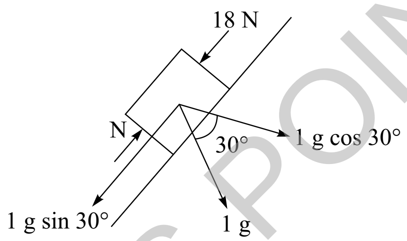
Net force along the incline:
T – mg sinθ = ma
Substitute:
30 – 5 × 10 × sin30° = 5a
30 – 25 = 5a ⇒ a = 1 m/s²

2. (b) Net acceleration of the system,

$a=\frac{\text{Net pulling force}}{M_{total}}$ (up the inclined)

$a=\frac{60-18-(3+2+1)\,g\sin 30°}{3+2+1}$

$=\frac{12}{6}=2$ m/s²



From F.B.D. of 1 kg

ma = N – 18 – 1 × 10 × $\frac{1}{2}$

⇒ 2 = N – 23 ⇒ N = 25 N

3. (a) Using equation of uniform motion,

$S_n=u+\frac{a}{2}(2n-1)$ where,

S_n is the distance covered in n^{th} sec
[from t = (n – 1) to t = n]
S_{n+1} is the distance covered in $(n+1)^{th}$ sec
[from t = n to t = (n + 1)]
Now; u = 0

$S_n=\frac{a}{2}(2n-1)$ … (i)

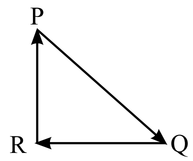
$S_{n+1}=\frac{a}{2}[2(n+1)-1]=\frac{a}{2}[2n+2-1]$

$S_{n+1}=\frac{a}{2}[2n+1]$ … (ii)

Dividing eqⁿ (i) and eqⁿ (ii)

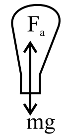
$\Rightarrow\frac{S_n}{S_{n+1}}=\frac{\left(\frac{a}{2}\right)(2n-1)}{\left(\frac{a}{2}\right)(2n+1)}\Rightarrow\frac{S_n}{S_{n+1}}=\frac{2n-1}{2n+1}$

4. (c) All these forces are forming closed loop in same order. So, net force is zero.
Force = ma



$\Rightarrow m\frac{d\vec{v}}{dt}=0\Rightarrow\vec{v}=$ constant

5. (a) For downward motion



$mg-F_a=ma\Rightarrow F_a=mg-ma$
If some mass Δm is removed, then it starts accelerating upwards
$F_a-(m-\Delta m)g=(m-\Delta m)a$
$mg-ma-mg+g\Delta m=ma-a\Delta m$
$g\Delta m-ma=ma-a\Delta m\Rightarrow\Delta m[g+a]=2ma$

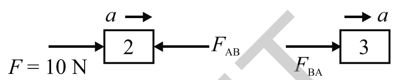
$\Delta m=\frac{2ma}{g+a}$

6. (a) In connected motion,

Common acceleration $a=\frac{F_{net}}{M_{total}}$

$a=\frac{10}{5}=2$ m/s²

Now, from FBD of 2 kg block & 3 kg block

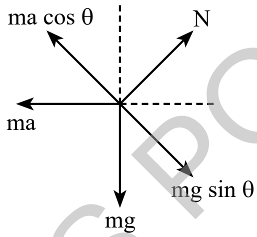


Force exerted by block A on block B,
$F_{BA}=(3)a$
$F_{BA}=3\times 2=6$ N

7. (b) $a=\frac{(m_2-m_1)g}{m_1+m_2}$

$a=\frac{(6-4)g}{6+4}=\frac{2g}{10}=\frac{g}{5}$

8. (d) From wedge reference frame



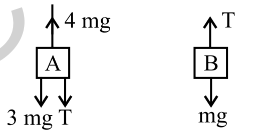
(FBD of mass m)

For block to be at rest

$ma\cos\theta = mg\sin\theta$

$a = g\tan\theta$

9. (d)



When the string is cut T = 0

for block A

$3\,m\,a = 4\,mg - 3\,mg$

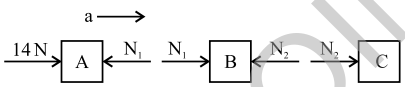
$a = \dfrac{g}{3}$

for block B

$mg = ma \Rightarrow a = g$

10. (a) Acceleration of system $= \dfrac{F_{net}}{M_{total}}$

$= \dfrac{14}{4+2+1} = 2\,ms^{-2}$



N_1 = Normal force between A and B

$14 - N_1 = m_A a$

$14 - N_1 = 4 \times 2$

$14 - N_1 = 8 \Rightarrow N_1 = 6\text{ N}$

11. (c)

$$\text{Acceleration} = \frac{\text{Net force in the direction of motion}}{\text{Total mass of system}}$$

$$= \frac{m_1 g - \mu(m_2 + m_3)g}{m_1 + m_2 + m_3}$$

$$= \frac{g(1-2\mu)}{3}\ [\because m_1 = m_2 = m_3]$$

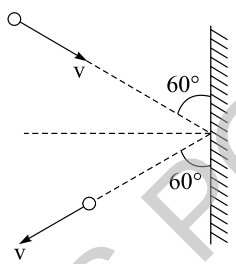
12. (b) As block of mass 2m moves with constant velocity so net force on it is zero.

13. (b) $\dfrac{1}{2}m\left(\dfrac{u}{3}\right)^2 - \dfrac{1}{2}mu^2 = -F_R \times 24$

$0 - \dfrac{1}{2}mu^2 = -F_R \times d$

$$\frac{\frac{1}{2}mu^2}{\frac{1}{2}mu^2 \times \frac{8}{9}} = \frac{d}{24} \Rightarrow d = 24 \times \frac{9}{8} = 27\,\text{cm}$$

14. (c)



The net force exerted on the wall,

$$F = \left|\frac{\Delta \vec{p}}{\Delta t}\right| = \frac{2mv\sin\theta}{t} = \frac{2(1)(1)\sin 60°}{0.1} = 10\sqrt{3}\text{ N}$$

15. (b) Momentum of the body,

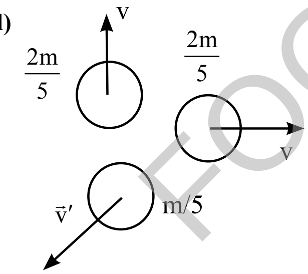
$= mv$

$34.3 = \dfrac{5}{1000} \times v \Rightarrow v = 60\text{ m/s}$

Distance travelled in 5 s = 60 × 5 = 300 m

16. (a) Centre of mass of rocket follows the same path not the fragments. It is because the explosion takes place due to internal forces.

17. (d)



As the shell was initially at rest, therefore initial momentum is zero.

Using conservation of momentum :

Initial momentum = Final momentum

$$0 = \frac{2m}{5}(-v\hat{i}) + \frac{2m}{5}(-v\hat{j}) + \frac{m}{5}\vec{v}'$$

$\Rightarrow \vec{v}' = 2v\hat{i} + 2v\hat{j}$

$\Rightarrow \vec{v}' = \sqrt{(2v)^2 + (2v)^2} = 2\sqrt{2}\,v$

18. (a) Given, mass of ball, m = 0.15 kg

Height from which it is dropped, h = 10 m

Velocity of ball just before reaching the ground, $v = \sqrt{2gh}$

$\Rightarrow v = \sqrt{2 \times 10 \times 10} = \sqrt{200} = 10\sqrt{2}\text{ m/s}$

According to the question, height remaining the same. So, velocity will also, only its direction changes.

i.e., $v_i = 10\sqrt{2}(-\hat{j})$ and $v_f = 10\sqrt{2}(\hat{j})$

Required magnitude of Impulse imported to the ball = change in linear momentum

$|\vec{I}| = |\Delta\vec{P}|$

$|\vec{I}| = |mv_f - mv_i|$

$= |mv_f - m(-v_f)| = |mv_f + mv_f|$

$= |2\,mv_f| = 2 \times 0.15 \times 10\sqrt{2}$

$\vec{I} = 3\sqrt{2}$ kg m/s $\Rightarrow \vec{I} = 4.2$ kg m/s

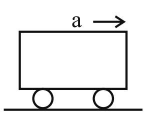
19. (c) Impulse $= |\overline{\Delta P}| = |\overline{m\Delta v}| = m(2v\cos 60°) = mv$

20. (c) Change in momentum,

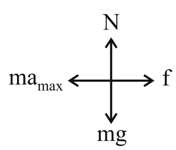
$\int \Delta p = \int F dt$ = Area of F – t graph

$= \dfrac{1}{2} \times 2 \times 6 - 3 \times 2 + 4 \times 3 = 12$ Ns

21. (d)



In the frame of car



$N = mg$

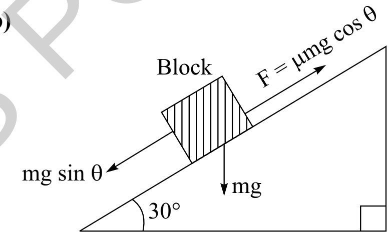
and $f = ma$

$f \le \mu N$

$\Rightarrow a \le \mu g \Rightarrow a \le 1.5\text{ ms}^{-2}$

or $a_{max} = 1.5\text{ ms}^{-2}$

22. (b)



From FBD $mg\sin\theta = \mu\,mg\cos\theta$

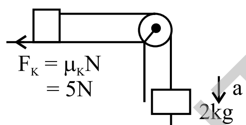
$\mu = \dfrac{1}{\sqrt{3}}$

$\tan 30° = \dfrac{1}{\sqrt{3}}$

as $\mu = \tan\theta$

The block is at rest and net force on it must be zero.

23. (d)



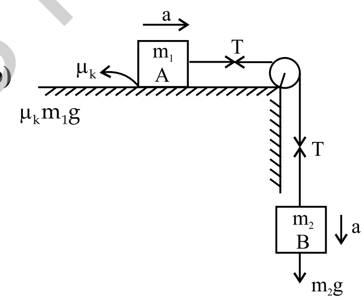
$a = \dfrac{20-5}{12} = 1.25\text{ m/s}^2$

24. (d) Coefficient of sliding friction is dimensionless.

$\because F = \mu N \Rightarrow \mu = \dfrac{F}{N}$

(F and N have same unit i.e., Newton)

25. (b)



For the motion of both blocks

$m_2 g - T = m_2 a$

$T - \mu_k m_1 g = m_1 a \Rightarrow a = \dfrac{(m_2 - \mu_k m_1)g}{m_1 + m_2}$

For the block of mass 'm_2'

$$m_2 g - T = m_2\left[\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right]g$$

$$T = m_2 g - \left[\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right]m_2 g = m_2 g\left[\frac{m_1 + \mu_k m_1}{m_1 + m_2}\right]$$

$$\Rightarrow T = \frac{m_1 m_2(1+\mu_k)g}{m_1 + m_2}$$

26. (c) Coefficient of static friction,

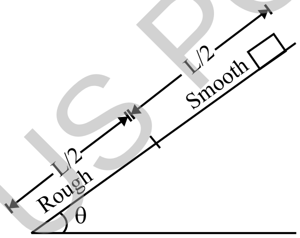
$\mu_s = \tan 30° = \dfrac{1}{\sqrt{3}} = 0.6$

$a = g\sin 30° - \mu_k g\cos 30°$

$S = ut + \frac{1}{2}at^2 \quad [\because u = 0]$

$\Rightarrow 4 = \frac{1}{2}\left[\frac{g}{2} - \frac{\mu_k g\sqrt{3}}{2}\right] \times 16 \Rightarrow \mu_k = 0.5$

27. (a)



Let m be mass of the block and L be length of the inclined plane.

For upper half smooth plane

Acceleration of the block, a = gsin θ

Here, u = 0 ($\because$ block starts from rest)

Using, $v^2 - u^2 = 2as$, we have

$v^2 - 0 = 2 \times g\sin\theta \times \frac{L}{2}$

$v = \sqrt{gL\sin\theta}$ … (*i*)

For lower half rough plane

Acceleration of the block, a' = gsin θ – μgcos θ where μ is the coefficient of friction between the block and lower half of the plane

Here, $u = v = \sqrt{gL\sin\theta}$

v = 0 ($\because$ block comes to rest)

a = a' = gsin θ – μgcos θ, $s = \frac{L}{2}$

Again, using $v^2 - u^2 = 2as$, we have

$0 - \left(\sqrt{gL\sin\theta}\right)^2 = 2 \times (g\sin\theta - \mu g\cos\theta) \times \frac{L}{2}$

– gLsin θ = (gsin θ – μg cos θ)L

– sin θ = sin θ – μcos θ

μ cos θ = 2sin θ

⇒ μ = 2tan θ

28. (a) In horizontal circular motion, the tension (*T*) in the string provides the centripetal force required to keep the bob moving in a circle. i.e.,

$T = m\omega^2 r$

For constant *m* and *r*, the tension is proportional to the square of the angular velocity:

$T \propto \omega^2$

Initial State:

Tension *T* corresponds to ω = 10 rpm.

After Quadrupling Tension (*T*' = 4T):

Since $\omega \propto \sqrt{T}$, the new angular velocity is:

$\omega' = \sqrt{4} \times \omega = 2 \times 10 = 20\,rpm$

29. (b) A particle moving with uniform speed in a circular path has varying velocity as the direction of motion changes continuously. The magnitude of velocity remains constant. As the direction of centripetal acceleration is changing, the particle has varying acceleration.

30. (d) $T = M\omega^2 r$

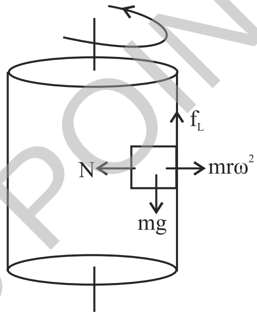
$T \propto \omega^2$

T becomes 4 times

31. (b) Both velocity and acceleration are vector quantity. So both magnitude and direction matters.

Direction of velocity and centripetal acceleration changes continuously. So $\vec{v}$ is not constant and $\vec{a}$ is not constant.

32. (c)



For equilibrium of the block limiting friction

$f_L \geq mg \Rightarrow \mu N \geq mg$

$\Rightarrow \mu m r\omega^2 \geq mg$

$\omega \geq \sqrt{\frac{g}{r\mu}}$

Therefore,

$\omega_{min} = \sqrt{\frac{g}{r\mu}}$

$\omega_{min} = \sqrt{\frac{10}{0.1 \times 1}} = 10\,\text{rad/s}$

33. (d) In uniform circular motion, tension provides the necessary centripetal force required to keep particle in motion.

So, net force on particle = T.

34. (b) We know that $\frac{v^2}{gR} = \left(\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right)$

$v = \sqrt{gR\left(\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right)}$

35. (b) $(F_C)_{heavier} = (F_C)_{lighter}$

$\Rightarrow \frac{2mV^2}{(r/2)} = \frac{m(nV)^2}{r} \Rightarrow n^2 = 4 \Rightarrow n = 2$

CHAPTER 5 Work, Energy and Power

Work Done

1. The potential energy of a particle moving along x-direction varies as $V = \dfrac{Ax^2}{\sqrt{x}+B}$. The dimensions of $\dfrac{A^2}{B}$ are: *(2024 Re)*

a. $[M^{3/2} L^{1/2} T^{-3}]$ b. $[M^{1/2} L\, T^{-3}]$
c. $[M^2 L^{1/2} T^{-4}]$ d. $[ML^2 T^{-4}]$

2. A force F = 20 + 10y acts on a particle in y direction where F is in newton and y in meter. Work done by this force to move the particle from y = 0 to y = 1 m is *(2019)*

a. 30 J b. 5 J c. 25 J d. 20 J

3. A particle moves from a point $\left(-2\hat{i}+5\hat{j}\right)$ to $\left(4\hat{j}+3\hat{k}\right)$ when a force of $\left(4\hat{i}+3\hat{j}\right)$ N is applied. How much work has been done by the force? *(2016 - II)*

a. 5 J b. 2 J c. 8 J d. 11 J

4. A uniform force of $\left(3\hat{i}+\hat{j}\right)$ newton acts on a particle of mass 2 kg. Hence the particle is displaced from position $\left(2\hat{i}+\hat{k}\right)$ metre to position $\left(4\hat{i}+3\hat{j}-\hat{k}\right)$ metre. The work done by the force on the particle is: *(2013)*

a. 15 J b. 9 J c. 6 J d. 13 J

Energy and Conservation of Mechanical Energy

5. An object falls from a height of 10 m above the ground. After striking the ground it loses 50% of its kinetic energy. The height upto which the object can rebounce from the ground is: *(2024 Re)*

a. 7.5 m b. 10 m c. 2.5 m d. 5 m

6. A bullet of mass m hits a block of mass M elastically. The transfer of energy is the maximum, then: *(2023-Manipur)*

a. M = m b. M = 2m c. M << m d. M >> m

7. A particle is released from height S from the surface of the Earth. At a certain height its kinetic energy is three times its potential energy. The height from the surface of earth and the speed of the particle at that **instant** are respectively: *(2021)*

a. $\dfrac{S}{4}, \dfrac{\sqrt{3gS}}{2}$ b. $\dfrac{S}{2}, \dfrac{\sqrt{3gS}}{2}$ c. $\dfrac{S}{4}, \sqrt{\dfrac{3gS}{2}}$ d. $\dfrac{S}{4}, \dfrac{3gS}{2}$

8. A body of mass (4m) is lying in x-y plane at rest. It suddenly explodes into three pieces. Two pieces each of mass (m) move perpendicular to each other with equal speeds (v). The total kinetic energy generated due to explosion is: *(2014)*

a. mv^2 b. $3/2mv^2$ c. $2\ mv^2$ d. $4\ mv^2$

9. An explosion breaks a rock into three parts in a horizontal plane. Two of them go off at right angles to each other. The first part of mass 1 kg moves with a speed of 12 ms^{-1} and the second part of mass 2 kg moves with 8 ms^{-1} speed. If the third part flies off with 4 ms^{-1} speed, then its mass is: *(2013, 2009)*

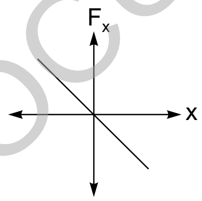
a. 17 kg b. 3 kg c. 5 kg d. 7 kg

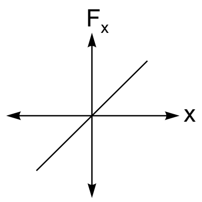
Spring

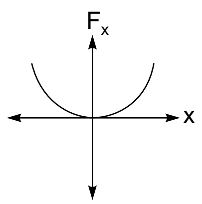
10. The potential energy of a long spring when stretched by 2 cm is U. If the spring is stretched by 8 cm, potential energy stored in it will be: *(2023)*

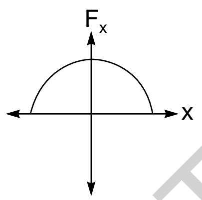
a. 16U b. 2U c. 4U d. 8U

11. The restoring force of a spring with a block attached to the free end of the spring is represented by *(2022 Re)*

a. 

b. 

c. 

d. 

12. Two similar springs P and Q have spring constants K_P and K_Q such that $K_P > K_Q$. They stretched first by the same amount (case a), then by the same force (case b). The work done by the springs W_P and W_Q are related as in case (a) and case (b), respectively: *(2015)*

a. $W_P = W_Q$; $W_P = W_Q$ b. $W_P > W_Q$; $W_Q > W_P$
c. $W_P < W_Q$; $W_Q < W_P$ d. $W_P = W_Q$; $W_P > W_Q$

Work Energy Theorem

13. An object moving along horizontal *x*-direction with kinetic energy 10 J is displaced through $x = (3\hat{i})m$ by the force $\vec{F} = (-2\hat{i} + 3\hat{j})$N. The kinetic energy of the object at the end of the displacement *x* is *(2024 Re)*

a. 10 J b. 16 J c. 4 J d. 6 J

14. Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take 'g' constant with a value 10 m/s^2. The work done by the (i) gravitational force and the (ii) resistive force of air is: *(2017-Delhi)*

a. (i) 1.25 J (ii) –8.25 J
b. (i) 100 J (ii) 8.75 J
c. (i) 10 J (ii) –8.75 J
d. (i) –10 J (ii) –8.25 J

15. A bullet of mass 10 g moving horizontally with a velocity of 400 ms^{-1} strikes a wooden block of mass 2 kg which is suspended by a light inextensible string of length 5 m. As a result, the center of gravity of the block is found to rise a vertical distance of 10 cm. The speed of the bullet after it emerges out horizontally from the block will be: *(2016 - II)*

a. 120 ms^{-1} b. 160 ms^{-1} c. 100 ms^{-1} d. 80 ms^{-1}

16. A block of mass 10 kg moving in x direction with a constant speed of 10 ms^{-1}, is subjected to a retarding force F = –0.1x J/m during its travel from x = 20 m to 30 m. Its final K.E. will be: *(2015)*

a. 450 J b. 275 J c. 250 J d. 475 J

Power

17. At any instant of time *t*, the displacement of any particle is given by 2*t* – 1 (SI unit) under the influence of force of 5 *N*. The value of instantaneous power is (in SI unit): *(2024)*

a. 7 b. 6 c. 10 d. 5

18. A particle moves with a velocity $\left(5\hat{i} - 3\hat{j} + 6\hat{k}\right)$ms^{-1} horizontally under the action of constant force $\left(10\hat{i} + 10\hat{j} + 20\hat{k}\right)$N. The instantaneous power supplied to the particle is: *(2023-Manipur)*

a. 200 W b. Zero c. 100 W d. 140 W

19. An electric lift with a maximum load of 2000 kg (lift + passengers) is moving up with a constant speed of 1.5ms^{-1}. The **frictional force** opposing the motion is 3000 N. The minimum power delivered by the motor to the lift in watts is : (g = 10 ms^{-2}) *(2022)*

a. 23500 b. 23000 c. 20000 d. 34500

20. The energy that will be ideally radiated by a 100 kW transmitter in **1 hour** is: *(2022)*

a. 1×10^5 J b. 36×10^7 J c. 36×10^4 J d. 36×10^5 J

21. Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional force are 10% of the **input energy**. How much power is generated by the turbine? (g = 10 m/s^2) *(2021, 2008)*

a. 8.1 kW b. 12.3 kW c. 7.0 kW d. 10.2 kW

22. A body of mass 1 kg begins to move under the action of a time dependent force $F = \left(2t\hat{i} + 3t^2\hat{j}\right)$ N, where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axis. What power will be developed by the force at the time t? *(2016 - I)*

a. $(2t^2 + 3t^2)$W b. $(2t^2 + 4t^4)$W
c. $(2t^3 + 4t^4)$W d. $(2t^3 + 3t^5)$W

23. A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest the force on the particle at time t is: *(2015)*

a. $\sqrt{mk}\,t^{\frac{-1}{2}}$ b. $\sqrt{2mk}\,t^{\frac{-1}{2}}$ c. $\frac{1}{2}\sqrt{mk}\,t^{\frac{-1}{2}}$ d. $\sqrt{\frac{mk}{2}t^{\frac{-1}{2}}}$

24. The heart of a man pumps 5 litres of blood through the arteries per minute at a pressure of 150 mm of mercury. If the density of mercury be 13.6×10^3 kg/m^3 and g = 10m/s^2, then the power of heart in watt is: *(2015 Re)*

a. 1.50 b. 1.70 c. 2.35 d. 3.0

Vertical Circle

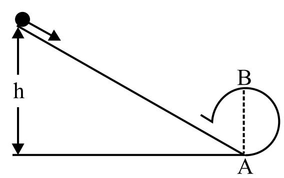
25. A point mass 'm' is moved in a vertical circle of radius 'r' with the help of a string. The velocity of the mass is $\sqrt{7gr}$ at the lowest point. The tension in the string at the **lowest point** is *(2020-Covid)*

a. 7 mg b. 8 mg c. 1 mg d. 6 mg

26. A mass m is attached to a thin wire and whirled in a vertical circle. The wire is most likely to break when: *(2019)*

a. The mass is at the highest point
b. The wire is horizontal
c. The mass is at the lowest point
d. Inclined at an angle of 60° from vertical

27. A body initially at rest and sliding along a frictionless track from a height h (as shown in the figure) just completes a vertical circle of diameter AB = D. The height h is equal to: *(2018)*



a. $\frac{7}{5}$D b. D c. $\frac{3}{2}$D d. $\frac{5}{4}$D

28. What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop? *(2016 - I)*

a. $\sqrt{gR}$ b. $\sqrt{2gR}$ c. $\sqrt{3gR}$ d. $\sqrt{5gR}$

29. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion? *(2016 - I)*

a. 0.1 m/s^2 b. 0.15 m/s^2 c. 0.18 m/s^2 d. 0.2 m/s^2

Collision

30. Two bodies A and B of same mass undergo completely inelastic one dimensional collision. The body A moves with velocity v_1 while body B is at rest before collision. The velocity of the system after collision is v_2. The ratio $v_1 : v_2$ is: *(2024)*

a. 4 : 1 b. 1 : 4 c. 1 : 2 d. 2 : 1

31. Body A of mass 4m moving with speed u collides with another body B of mass 2m, at rest. The collision is head on and elastic in nature. After the collision the fraction of energy lost by the colliding **body A** is: *(2019)*

a. $\frac{1}{9}$ b. $\frac{8}{9}$ c. $\frac{4}{9}$ d. $\frac{5}{9}$

32. A moving block having mass m, collides with another stationary block having mass 4m. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution(e) will be *(2018)*

a. 0.8 b. 0.25 c. 0.5 d. 0.4

33. Two identical balls A and B having velocities of 0.5 m/s and –0.3 m/s respectively collide elastically in one dimension. The velocities of B and A **after** the collision respectively will be: *(2016, 1998, 1994, 1991)*

a. –0.3 m/s and 0.5 m/s b. 0.3 m/s and 0.5 m/s
c. –0.5 m/s and 0.3 m/s d. 0.5 m/s and –0.3 m/s

34. Two particles of masses m_1, m_2 move with initial velocities u_1 and u_2. On collision, one of the particles get excited to higher level, after absorbing energy ε. If final velocities of particles be v_1 and v_2, then we must have: *(2015)*

a. $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \varepsilon$

b. $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

c. $\frac{1}{2}m_1^2u_1^2 + \frac{1}{2}m_2^2u_2^2 + \varepsilon = \frac{1}{2}m_1^2v_1^2 + \frac{1}{2}m_2^2v_2^2$

d. $m_1^2u_1 + m_2^2u_2 - \varepsilon = m_1^2v_1 + m_2^2v_2$

35. On a frictionless surface, a block of mass M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed v/3. The second block's speed after the collision is: *(2015 Re)*

a. $\frac{\sqrt{3}}{2}v$ b. $\frac{2\sqrt{2}}{3}v$ c. $\frac{3}{4}v$ d. $\frac{3}{\sqrt{2}}v$

36. A ball is thrown vertically downwards from a height of 20 m with an initial velocity u_0 It collides with the ground, loses 50 percent of its energy in collision and rebounds to the same height. The initial velocity u_0 is: (Take g = 10 ms^{-2}) *(2015 Re)*

a. 10 m/s b. 14 m/s c. 20 m/s d. 28 m/s

37. Two particles A and B, move with constant motion in one dimensional with velocities $\vec{v}_1$ and $\vec{v}_2$. At the initial moment their position vectors are $\vec{r}_1$ and $\vec{r}_2$ respectively. The condition for particle A and B for their collision is: *(2015 Re)*

a. $\vec{r}_1 - \vec{r}_2 = \vec{v}_1 - \vec{v}_2$ b. $\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$

c. $\vec{r}_1 . \vec{v}_1 = \vec{r}_2 . \vec{v}_2$ d. $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$

Answer Key

1. (c)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (a)
11. (a)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (a)	**16.** (d)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (b)
21. (a)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (d)	**28.** (d)	**29.** (a)	**30.** (d)
31. (b)	**32.** (b)	**33.** (d)	**34.** (b)	**35.** (b)	**36.** (c)	**37.** (b)			

Explanations

1. (c) Given:

Potential energy $V = \dfrac{Ax^2}{\sqrt{x+B}}$.

For dimensional consistency:

$[V] = [\text{Energy}] = ML^2T^{-2}$

Denominator $\sqrt{x+B}$

$\Rightarrow [B] = [x] = L^{1/2}$.

Thus:

$$[A] = \frac{[V]\sqrt{L^{1/2}}}{L^2} = ML^{1/2}T^{-2}$$

$$\frac{A^2}{B} = \frac{\left(ML^{1/2}T^{-2}\right)^2}{L^{1/2}} = M^2L^{1/2}T^{-4}$$

2. (c) Work done by variable force is

$$W = \int_{y_i}^{y_f} F\,dy$$

Here, $y_i = 0$, $y_f = 1$ m

$$\therefore W = \int_0^1 (20+10y)\,dy = \left[20y + 5y^2\right]_0^1$$

$= 20 (1 - 0) + 5 (1 - 0) = 25j$

3. (a) $\vec{s} = \vec{r}_f - \vec{r}_i = 2\hat{i} - \hat{j} + 3\hat{k}$

$W = \vec{F}.\vec{s} = \left(4\hat{i} + 3\hat{j}\right).\left[2\hat{i} - \hat{j} + 3\hat{k}\right] = 8 - 3 = 5$ J

4. (b)

$W = \vec{F}.\vec{S} = \left(3\hat{i} + \hat{j}\right).\left[(4-2)\hat{i} + (3-0)\hat{j} + (-1-1)K\right]$

$= \left(3\hat{i} + \hat{j}\right).\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right) = 3(2) + 1(3) + 0(-2) = 9$ J

5. (d) Before hitting the ground,

$K.E_1 = mg \times 10$

Just after hitting the ground

$$K.E_2 = K.E_1 - 0.5\ K.E_1 = \frac{K.E_1}{2}$$

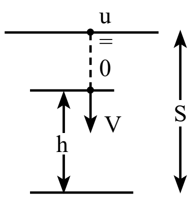
$\therefore K.E_2 = mgh_2$

$$\frac{K.E_1}{2} = mgh_2 \Rightarrow \frac{mg\times 10}{2} = mgh_2$$

$h_2 = 5$ m

6. (a) If M = *m* then maximum amount of energy is transferred.

7. (c) Required figure,



Particle is released, i.e., its initial velocity is zero, u = 0 m/s.

Let the height at any instant be 'h'.

Total energy remains conserved at any point.

$\Rightarrow$ K.E + P.E = E_{Total}

$\Rightarrow$ 3 P.E + P.E = E_{Total}

$\Rightarrow$ 4 P.E = E_{Total}

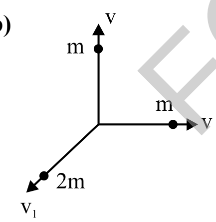
$\Rightarrow 4(mgh) = mgS \Rightarrow h = \dfrac{S}{4}$

Velocity of a particle under free fall is given by;

$V = \sqrt{2gh}$

$$V = \sqrt{2g\left(S - \frac{S}{4}\right)} = \sqrt{\frac{3gS}{2}}$$

8. (b)



Net momentum of equal masses = $\sqrt{2}mv$

By conservation of linear momentum

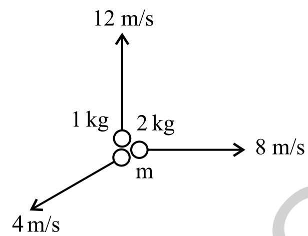
$$2mv_1 = \sqrt{2}mv \Rightarrow v_1 = \frac{v}{\sqrt{2}}$$

Total K.E generated

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4}(2m)v^2$$

$$= mv^2 + \frac{mv^2}{2} = \frac{3}{2}mv^2$$

9. (c)



From conservation of momentum

$$m(4) = \sqrt{(1\times 12)^2 + (2\times 8)^2} \Rightarrow m = 5\text{ kg}$$

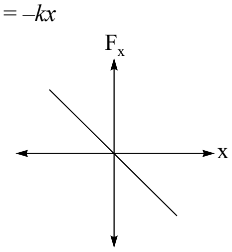
10. (a) Potential energy stored in the spring $= \dfrac{1}{2}kx^2$

Now $\dfrac{1}{2}k(2)^2 = U$

$\& \dfrac{1}{2}k(8)^2 = U'$ (say)

$\Rightarrow U' = \dfrac{64}{4}U = 16\,U$

11. (a) F = –*kx*



12. (b) Given $K_P > K_Q$

Case (a): $x_1 = x_2 = x$

$$\frac{W_P}{W_Q} = \frac{\frac{1}{2}K_Px^2}{\frac{1}{2}K_Qx^2} = \frac{K_P}{K_Q} \Rightarrow W_P > W_Q$$

Case (b): $F_1 = F_2 = F$

For constant force

$$W = \frac{F^2}{2K} \Rightarrow W \propto \frac{1}{K}$$

So, $\dfrac{W_P}{W_Q} = \dfrac{K_Q}{K_P} \Rightarrow W_Q > W_P$

13. (c) Work done:

$W = \vec{F}\cdot\Delta\vec{x} = (-2)(3) + (3)(0) = -6\,J.$

From work energy theorem:

$K_f = K_i + W = 10\text{ J} - 6\text{ J} = 4$ J.

14. (c) Work done by gravitational force $= W_g = mgh$

$= 10^{-3} \times 10 \times 10^3 = 10$ J

Apply work energy theorem

$W_{\text{all force}} = K_f - K_i$

$W_{(\text{conservative})} + W_{(\text{non conservative})} = K_f - K_i$

$W_g + W_{fr} = \dfrac{1}{2}mv^2 - 0\ [\because K_i = 0]$

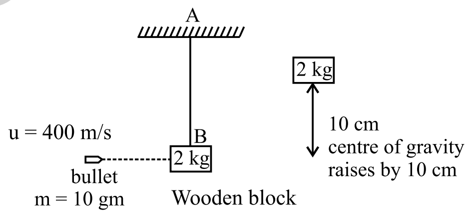
$\Rightarrow mgh + W_{fr} = \dfrac{1}{2}mv^2$

$\Rightarrow 10^{-3}\times 10 \times 1000 + W_{fr} = \dfrac{1}{2}\times 10^{-3}\times (50)^2$

$\Rightarrow 10 + W_{fr} = 1.25$

$\therefore$ Work done by air resistance = –8.75 J

15. (a) AB = 5 m



Apply conservation of linear momentum

mu + 0 = mv + MV

$$\frac{10}{1000}\times 400 + 0 = \frac{10}{1000}v + 2V$$

0.01v + 2V = 4 … (*i*)

According to conservation of energy

PE at max height = KE at bottom

$$MgH = \frac{1}{2}\times MV^2$$

$$2\times 10\times\frac{10}{100} = \frac{1}{2}\times 2\times V^2 \Rightarrow V^2 = 2$$

$V = \sqrt{2}\ ms^{-1}$

Substituting the value of V in Eq. (*i*), we get

$\frac{v}{100}+2\sqrt{2}=4 \Rightarrow v=\left(4-2\sqrt{2}\right)100$

$\simeq 120\ ms^{-1}$

16. (d) $F = -0.1\ x\ J/m$

According to Work Energy theorem

Work done by all force $= K_f - K_i$

$\Rightarrow \int F.dx = K_f - K_i$

$\Rightarrow \int_{20}^{30}(-0.1x)\ dx = K_f - \frac{1}{2}\times mu^2$

$(-)0.1\left[\frac{x^2}{2}\right]_{20}^{30} = K_f - \frac{1}{2}\times 10\times 10^2$

$\frac{1}{10\times 2}\left[x^2\right]_{30}^{20} = K_f - 500$

$\frac{1}{20}\times[400-900] = K_f - 500$

$-\frac{500}{20} = K_f - 500$

$K_f = 500 - 25 = 475\ J$

17. (c) $s = 2t - 1$

$v = \frac{ds}{dt} = \frac{d}{dt}(2t-1) = 2$ (SI unit)

Instantaneous power, $P = \vec{F}.\vec{v}$

$= 5 \times 2 = 10$ (SI unit)

18. (d) Instantaneous power of the particle,

$P = \vec{F}.\vec{V}$

$P = (10\hat{i}+10\hat{j}+20\hat{k}).(5\hat{i}-3\hat{j}+6\hat{k})$

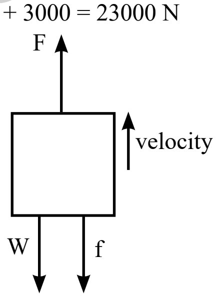
$P = 50 - 30 + 120$

$P = 140\ W$

19. (d) As the velocity is constant, therefore net upward force is equal to the net downward force

$\Rightarrow F = W + f$

$= 20000 + 3000 = 23000\ N$



As, Power $= \vec{F}\cdot\vec{v}$

$= 23000 \times 1.5 = 34500\ W$

20. (b) Energy = Power × time

$= 100 \times 10^3 \times 3600 = 36 \times 10^7\ J$

21. (a) Input power, $P_{in} = \frac{d(mgh)}{dt}$

$P_{in} = gh\left(\frac{dm}{dt}\right)$

$P_{in} = gh(15) = 10 \times 60 \times 15 = 9000$ watt

Again,

$P_{out} = \left(\frac{100-10}{100}\right)$ of P_{in}

$= \frac{90}{100}\times 9000 = 8100$ watt $= 8.1$ kW

22. (d)

$\vec{F} = 2t\hat{i} + 3t^2\hat{j}$

$m\frac{d\vec{v}}{dt} = 2t\hat{i} + 3t^2\hat{j}$

$\int_0^{\vec{v}} d\vec{V} = \int_0^t\left(2t\hat{i}+3t^2\hat{j}\right)dt = t^2\hat{i}+t^3\hat{j}$

Power $= \vec{F}.\vec{V} = \left(2t\hat{i}+3t^2\hat{j}\right)\cdot\left(t^2\hat{i}+t^3\hat{j}\right) = \left(2t^3+3t^5\right)W$

23. (d) $P = Fv = mav \Rightarrow k = mv\frac{dv}{dt}$

$\Rightarrow vdv = \frac{k}{m}dt$

By integrating the equation

$\int vdv = \int\frac{k}{m}dt$

$\Rightarrow \frac{v^2}{2} = \frac{k}{m}t \Rightarrow v = \sqrt{\frac{2kt}{m}}$

$a = \frac{dv}{dt} = \sqrt{\frac{2k}{m}}\left(\frac{1}{2}t^{-1/2}\right)$

$F = ma = m\left(\frac{1}{2}\right)\sqrt{\frac{2k}{mt}} \Rightarrow F = \sqrt{\frac{mk}{2t}} = \sqrt{\frac{mk}{2}}\,t^{\frac{-1}{2}}$

24. (b) Pressure = 150 mm Hg

Pumping rate $= \frac{dV}{dt} = \frac{5\times 10^{-3}}{60}\ m^3/s$

Power of heart $= P.\frac{dV}{dt} = \rho gh\times\frac{dV}{dt}$

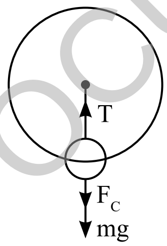
$= \left(13.6\times 10^3\ kg/m^3\right)(10)\times(0.15)\times\frac{5\times 10^{-3}}{60}$

$= \frac{13.6\times 5\times 0.15}{6} = 1.72$ watt

25. (b) According to the given question

$T - mg = F_c$

(centrifugal force)



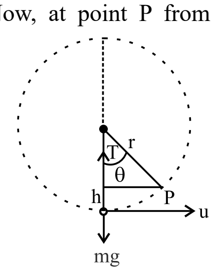
$T - mg = m\frac{\left(\sqrt{7gr}\right)^2}{r}$

$\Rightarrow T = 8\ mg$

26. (c) Let the mass m makes a small angle 'θ' with vertical Now, at point P from figure

$T - mg\cos\theta = \frac{mV^2}{r}$

$\therefore T = mg\cos\theta + \frac{mv^2}{r}$



From figure, $\cos\theta = \frac{r-h}{r}$

So T will be maximum when cos θ will be 1

$\therefore \frac{r-h}{r} = 1 \Rightarrow h = 0$

The tension is maximum when the mass is at the lowest position of the verticle circle, so the chance of breaking is maximum.

27. (d) To complete verticle circle minimum velocity required at lowest point of circle is $\sqrt{5gr}$. So by conservation of energy.

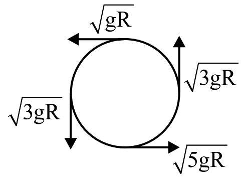
$mgh = \frac{1}{2}mv^2$

here, $V = \sqrt{5g\times\frac{D}{2}}$

$\therefore mgh = \frac{1}{2}m\times 5g\frac{D}{2}$

$h = \frac{5D}{4}$

28. (d) Minimum velocity required at different points to complete full vertical circle



29. (a) $\frac{1}{2}mv^2 = E \Rightarrow \frac{1}{2}\left(\frac{10}{1000}\right)v^2 = 8\times 10^{-4}$

$v^2 = \left(8\times 10^{-4}\right)200 = \frac{16}{100}ms^{-1}$

$v = \frac{4}{10}ms^{-1}$

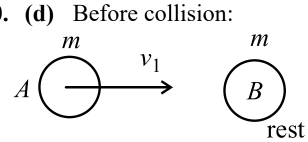
Now applying $v^2 - u^2 = 2as$

$\left(\frac{4}{10}\right)^2 = 2a(4\pi R); \left[s = 2(2\pi R) = 4\pi R\right]$

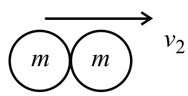
$\Rightarrow \frac{16}{100} = 2a\left(4\pi\frac{6.4}{100}\right)$

$\therefore a = \frac{16}{100}\times\left[\frac{7\times 100}{8\times 22\times 6.4}\right] = 0.1\ m/s^2$

30. (d) Before collision:



After collision:



Now, from conservations of momentum,

Momentum before collision = Momentum after collision.

$mv_1 + (0) = 2m\,v_2$

$mv_1 = 2mv_2$

$\frac{v_1}{v_2} = \frac{2}{1}$

$\therefore v_1 : v_2 = 2 : 1$

31. (b) From law of conservation of momentum we have

$m_1u_1 = m_1v_1 + m_2v_2$

$m_2v_2 = m_1u_1 - m_1v_1 = m_1(u_1 - v_1)$ ... (*i*)

From the law of conservation of K.E. we have

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow m_2v_2^2 = m_1(u_1^2 - v_1^2) \quad \ldots (ii)$$

Dividing (*ii*) with (*i*)

$$v_2 = u_1 + v_1 \quad \ldots (iii)$$

eliminating v_2 from (*i*) and (*iii*) we get

$$\frac{m_1u_1 - m_1v_1}{m_2} = u_1 + v_1$$

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1$$

$$\Rightarrow \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 = v_1$$

Fraction of KE of m_1 carried by m_2 is

$$\frac{\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2}{\frac{1}{2}m_1v_1^2}$$

$$= 1 - \left[\frac{m_1 - m_2}{m_1 + m_2}\right]^2 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

This is also equal to the fractional transfer of KE of colliding body.

Now,

$$\frac{\Delta KE}{KE} = \frac{4(m_1m_2)}{(m_1+m_2)^2} = \frac{4(4m)2m}{(4m+2m)^2} = \frac{32m^2}{36m^2} = \frac{8}{9}$$

This fractional transfer is equal to the fraction of energy lost by A.

32. **(b)** Since there is no external force on the system, so momentum of the system will remain constant.

$$mv = 4mv' \Rightarrow v' = \frac{v}{4}$$

$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

$$\therefore e = \frac{v'}{v} = \frac{v/4}{v} = \frac{1}{4} = 0.25$$

33. **(d)** Since both bodies are identical and collision is elastic. Therefore velocities will be interchanged after collision.

$v_A = -0.3$ m/s and $v_B = 0.5$ m/s

34. **(b)** Energy will always be conserved so

K. E.$_{\text{initial}}$ = K.E.$_{\text{final}}$ + Excitation energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \varepsilon$$

35. **(b)** In elastic collision energy of system remains same so

$$(K.E)_{\text{before collision}} = (K.E)_{\text{after collision}}$$

Let speed of second body after collision is V

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}m\left(\frac{v}{3}\right)^2 + \frac{1}{2}m(v')^2 \Rightarrow v' = \frac{2\sqrt{2}}{3}v$$

36. **(c)** Let ball rebounds with speed V so

$$v = \sqrt{2gh} = \sqrt{2\times10\times20} = 20 \text{ m/s}$$

Energy just after rebound

$$E = \frac{1}{2}\times m\times v^2 = 200m$$

50% energy loses in collision means just before collision energy is 400m

By using energy conservation

$$\frac{1}{2}mv_0^2 + mgh = 400m$$

$$\Rightarrow \frac{1}{2}mv_0^2 + m\times10\times20 = 400m$$

$$\Rightarrow v_0 = 20 \text{ ms}^{-1}$$

37. **(b)** For the collision, the final position of both the particle should be same

$$\therefore \vec{r_1} + \vec{v_1}t = \vec{r_1} + \vec{v_2}t$$

$$\Rightarrow \vec{r_1} - \vec{v_2}t = (\vec{v_2} - \vec{v_1})t$$

$$\Rightarrow \vec{r_1} - \vec{r_2} = (\vec{v_2} - \vec{v_1})\frac{|\vec{r_1} - \vec{r_2}|}{|\vec{v_2} - \vec{v_1}|}$$

$$\Rightarrow \frac{\vec{r_1} - \vec{r_2}}{|\vec{r_1} - \vec{r_2}|} = \frac{\vec{v_2} - \vec{v_1}}{|\vec{v_2} - \vec{v_1}|}$$

CHAPTER 6

System of Particles and Rotational Motion

Centre of Mass and Motion of Centre of Mass

1. Two particles A and B initially at rest, move towards each other under mutual force of attraction. At an instance when the speed of A is v and speed of B is $3v$, the speed of centre of mass is: *(2023-Manipur)*

a. $2v$ b. zero c. v d. $4v$

2. Two objects of mass 10 kg and 20 kg respectively are connected to the two ends of a rigid rod of length 10 m with negligible mass. The distance of the centre of mass of the system **from the 10 kg mass** is : *(2022)*

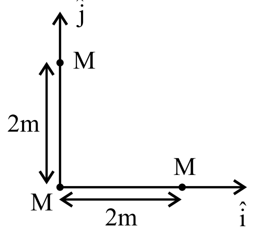
a. 5m b. $\frac{10}{3}$m c. $\frac{20}{3}$m d. 10 m

3. Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass.

The centre of mass of the system from the 5 kg particle is nearly at a distance of : *(2020)*

a. 50 cm b. 67 cm c. 80 cm d. 33 cm

4. Three identical spheres, each of mass M, are placed at the corners of a right angle triangle with mutually perpendicular sides equal to 2 m (see figure). Taking the point of intersection of the two mutually perpendicular sides as the **origin**, find the position vector of centre of mass. *(2020-Covid)*



a. $(\hat{i}+\hat{j})$ b. $\frac{2}{3}(\hat{i}+\hat{j})$ c. $\frac{4}{3}(\hat{i}+\hat{j})$ d. $2(\hat{i}+\hat{j})$

5. Which of the following statements are correct?

A. Centre of mass of a body always coincides with the centre of gravity of the body

B. Centre of gravity of a body is the point at which the total gravitational torque on the body is zero

C. A couple on a body produce both translational and rotational motion in a body.

D. Mechanical advantage greater than one means that small effort can be used to lift a large load *(2017-Delhi)*

a. A and B b. B and C c. C and D d. B and D

6. Two spherical bodies of mass M and 5M and radii R and 2R are released in free space with initial separation between their centers equal to 12R. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is: *(2015)*

a. 4.5R b. 7.5R c. 1.5R d. 2.5R

Angular Displacement, Velocity and Acceleration

7. Let ω_1, ω_2 and ω_3 be the angular speed of the second hand, minute hand and hour hand of a smoothly running analog clock, respectively. If x_1, x_2 and x_3 are their respective angular distances in 1 minute then the factor which remains constant (k) is *(2024 Re)*

a. $\frac{\omega_1}{x_1}=\frac{\omega_2}{x_2}=\frac{\omega_3}{x_3}=k$

b. $\omega_1 x_1=\omega_2 x_2=\omega_3 x_3=k$

c. $\omega_1 x_1^2=\omega_2 x_2^2=\omega_3 x_3^2=k$

d. $\omega_1^2 x_1=\omega_2^2 x_2=\omega_3^2 x_3=k$

8. The angular acceleration of a body, moving along the circumference of a circle, is: *(2023)*

a. along the axis of rotation

b. along the radius, away from centre

c. along the radius towards the centre

d. along the tangent to its position

9. The angular speed of a fly wheel moving with uniform angular acceleration changes from 1200 rpm to 3120 rpm in 16 seconds. The angular acceleration in rad/s^2 is : *(2022)*

a. 104π b. 2π c. 4π d. 12π

10. The angular speed of the wheel of a vehicle is increased from 360 rpm to 1200 rpm in 14 second. Its angular acceleration is. *(2020-Covid)*

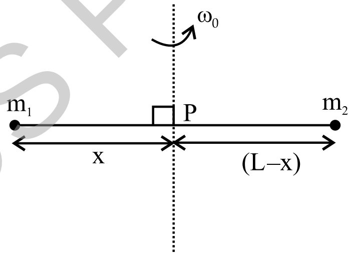
a. 28π rad/s^2 b. 120π rad/s^2

c. 1 rad/s^2 d. 2π rad/s^2

11. A uniform circular disc of radius 50 cm at **rest** is free to turn about an axis which is perpendicular to its plane and passes through its center. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2}. Its net acceleration in ms^{-2} at the end of 2.0 s is approximately: *(2016 - I)*

a. 8.0 b. 7.0 c. 6.0 d. 3.0

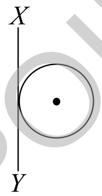
12. Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L, and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point P on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity ω_0 is minimum, is given by: *(2015 Re)*



a. $x = \frac{m_2 L}{m_1 + m_2}$ b. $x = \frac{m_1 L}{m_1 + m_2}$
c. $x = \frac{m_1}{m_2} L$ d. $x = \frac{m_2}{m_1} L$

Moment of Inertia, Theorem of Parallel and Perpendicular Axis and Energy in Rotation

13. The radius of gyration of a solid sphere of mass 5 kg about *XY* is 5 m as shown in figure. The radius of the sphere is $\frac{5x}{\sqrt{7}}$ m, then the value of *x* is: *(2024 Re)*



a. 5 b. $\sqrt{2}$ c. $\sqrt{3}$ d. $\sqrt{5}$

14. The moment of inertia of a thin rod about an axis passing through its mid point and perpendicular to the rod is 2400 g cm^2. The length of the 400 g rod is nearly: *(2024)*
a. 20.7 cm b. 72.0 cm c. 8.5 cm d. 17.5 cm

15. The ratio of radius of gyration of a solid sphere of mass M and radius R about its own axis to the radius of gyration of the thin hollow sphere of same mass and radius about its axis is: *(2023)*
a. 5 : 2 b. 3 : 5 c. 5 : 3 d. 2 : 5

16. An energy of 484 J is spent in increasing the speed of a flywheel from 60 rpm to 360 rpm.
The moment of inertia of the flywheel is *(2022 Re)*
a. 0.07 kg-m^2 b. 0.7 kg-m^2
c. 3.22 kg-m^2 d. 30.8 kg-m^2

17. The ratio of the radius of gyration of a thin uniform disc about an axis passing through it centre and normal to its plane to the radius of gyration of the disc about its diameter is: *(2022)*
a. $1:\sqrt{2}$ b. 2 : 1 c. $\sqrt{2}:1$ d. 4 : 1

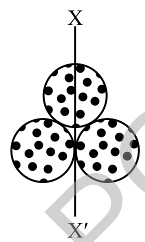
18. From a circular ring of mass 'M' and radius 'R' an arc corresponding to a 90° sector is removed. The moment of inertia of the **remaining part** of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is 'K' times 'MR^2'. Then the value of 'K' is: *(2021)*
a. $\frac{7}{8}$ b. $\frac{1}{4}$ c. $\frac{1}{8}$ d. $\frac{3}{4}$

19. Three objects, A: (a solid sphere), B: (a thin circular disk) and C: (a circular ring), each have the same mass M and radius R. They all spin with the same angular speed ω about their own symmetry axes. The amounts of work (W) required to bring them to **rest**, would satisfy the relation *(2018)*
a. $W_B > W_A > W_C$ b. $W_A > W_B > W_C$
c. $W_C > W_B > W_A$ d. $W_A > W_C > W_B$

20. A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation (E_{sphere} / $E_{cylinder}$) will be: *(2016 - II)*
a. 1 : 4 b. 3 : 1 c. 2 : 3 d. 1 : 5

21. From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre? *(2016 - I)*
a. $15MR^2/32$ b. $13MR^2/32$
c. $11MR^2/32$ d. $9MR^2/32$

22. Three identical spherical shells, each of mass *m* and radius *r* are placed as shown in figure. Consider an axis *XX′* which is touching to two shells and passing through diameter to third shell. Moment of inertia of the system consisting of these three spherical shells about *XX′* axis is: *(2015)*

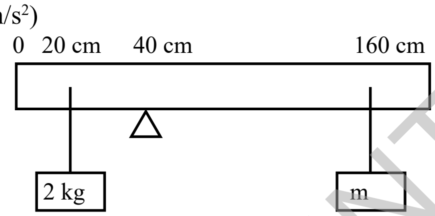


a. $3mr^2$ b. $16/5mr^2$ c. $4mr^2$ d. $11/5\ mr^2$

Torque, Angular Momentum and its Conservation

23. A constant torque of 100 N m turns a wheel of moment of inertia 300 kg m^2 about an axis passing through its centre. Starting from rest, its angular velocity after 3s is: *(2023-Manipur)*
a. 1 rad/s b. 5 rad/s c. 10 rad/s d. 15 rad/s

24. A uniform rod of length 200 cm and mass 500 g is balanced on a wedge placed at 40 cm mark. A mass of 2 kg is suspended from the rod at 20 cm and another unknown mass 'm' is suspended from the rod at 160 cm mark as shown in the figure. Find the value of 'm' such that the rod is in **equilibrium**. ($g = 10\ m/s^2$) *(2021)*



a. $\frac{1}{3}$ kg b. $\frac{1}{6}$ kg c. $\frac{1}{12}$ kg d. $\frac{1}{2}$ kg

25. Find the torque about the origin when a force of $3\hat{j}$ N acts on a particle whose position vector is $2\hat{k}$ m. *(2020)*
a. $6\hat{j}$ N m b. $-6\hat{i}$ N m c. $6\hat{k}$ N m d. $6\hat{i}$ N m

26. A solid cylinder of mass 2 kg and radius 4 cm is rotating about its axis at the rate of 3 rpm. The torque required to stop after 2π revolutions is *(2019)*

a. 2×10^{-6} N m b. 2×10^{-3} N m

c. 12×10^{-4} N m d. 2×10^{6} N m

27. A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain **constant** for the sphere? *(2018)*

a. Rotational kinetic energy b. Moment of inertia

c. Angular velocity d. Angular momentum

28. The moment of the force, $\vec{F} = 4\hat{i} + 5\hat{j} - 6\hat{k}$ at (2, 0, –3), about the point (2, –2, –2) is given by *(2018)*

a. $-7\hat{i} - 8\hat{j} - 4\hat{k}$ b. $-4\hat{i} - \hat{j} - 8\hat{k}$

c. $-8\hat{i} - 4\hat{j} - 7\hat{k}$ d. $-7\hat{i} - 4\hat{j} - 8\hat{k}$

29. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? *(2017-Delhi)*

a. 0.25 rad/s^2 b. 25 rad/s^2

c. 5 m/s^2 d. 25 m/s^2

30. Two rotating bodies A and B of masses m and 2m with moments of inertia I_A and I_B ($I_B > I_A$) have equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then: *(2016 - II)*

a. $L_B > L_A$ b. $L_A > L_B$ c. $L_A = \frac{L_B}{2}$ d. $L_A = 2L_B$

31. An automobile moves on a road with a speed of 54 km/h. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is 3 kgm^2. If the vehicle is brought to rest in 15 s, the magnitude of average torque transmitted by its brakes to wheel is: *(2015 Re)*

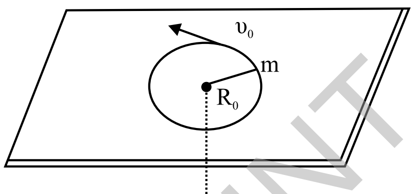
a. 2.86 kg m^2/s^2 b. 6.66 kg m^2/s^2

c. 8.58 kg m^2/s^2 d. 10.86 kg m^2/s^2

32. A force $\vec{F} = \alpha\hat{i} + 3\hat{j} + 9\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$. The value of α for which angular momentum about origin is conserved is: *(2015 Re)*

a. 1 b. –1 c. 2 d. Zero

33. A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to a string which passes through a smooth hole in the plane as shown.



The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is: *(2015)*

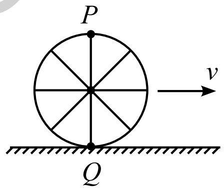
a. $\frac{1}{4}mv_0^2$ b. $2mv_0^2$ c. $\frac{1}{2}mv_0^2$ d. mv_0^2

34. A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 rev/s^{-2} is: *(2014)*

a. 25 N b. 50 N c. 78.5 N d. 157 N

Rolling Motion

35. A wheel of a bullock cart is rolling on a level road as shown in the figure below. If its linear speed is v in the direction shown, which one of the following options is correct (*P* and *Q* are any highest and lowest points on the wheel, respectively)? *(2024)*



a. Both the points *P* and *Q* move with equal speed.

b. Point *P* has zero speed.

c. Point *P* moves slower than point *Q*.

d. Point *P* moves faster than point *Q*.

36. A disc of radius 2 m and mass 100 kg rolls on a horizontal floor. Its centre of mass has speed of 20 cm/s. How much work is needed to stop it? *(2019)*

a. 3 J b. 30 kJ c. 2 J d. 1 J

37. A solid sphere is in rolling motion. In rolling motion a body possesses translational kinetic energy (K_t) as well as rotational kinetic energy (K_r) simultaneously. The ratio K_t: (K_t+ K_r) for the sphere is: *(2018)*

a. 10 : 7 b. 5 : 7 c. 7 : 10 d. 2 : 5

38. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is: *(2017-Delhi)*

a. $\frac{1}{4}I(\omega_1 - \omega_2)^2$ b. $I(\omega_1 - \omega_2)^2$

c. $\frac{1}{8}I(\omega_1 - \omega_2)^2$ d. $\frac{1}{2}I(\omega_1 - \omega_2)^2$

39. A disk and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first? *(2016 - I)*

a. Disk

b. Sphere

c. Both reach at the same time

d. Depends on their masses

40. The ratio of the accelerations for a solid sphere (mass m and radius R) rolling down an incline of angle 'θ' without slipping and slipping down the incline without rolling is: *(2014)*

a. 5 : 7 b. 2 : 3 c. 2 : 5 d. 7 : 5

41. Small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is: *(2013)*

a. Disc b. Ring

c. Solid sphere d. Hollow sphere

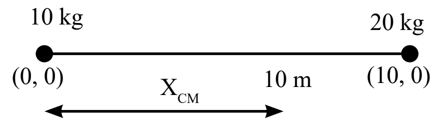
Answer Key

1. (b)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (d)
11. (a)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (None)	**16.** (b)	**17.** (c)	**18.** (d)	**19.** (c)	**20.** (d)
21. (b)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (a)	**27.** (d)	**28.** (d)	**29.** (b)	**30.** (a)
31. (b)	**32.** (b)	**33.** (b)	**34.** (d)	**35.** (d)	**36.** (a)	**37.** (b)	**38.** (a)	**39.** (b)	**40.** (a)
41. (a)									

Explanations

1. (b) Since net external force on system is zero. Final velocity of centre of mass = Initial velocity of centre of mass = 0.

2. (c)



$\because \quad X_{CM} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

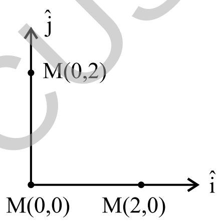
$\Rightarrow \quad X_{CM} = \dfrac{20\times10}{20+10} = \dfrac{20}{3}\,m$

3. (b) Here, we have two masses let $m_1 = 5$ kg, $m_2 = 10$ kg. For two bodies system

By using $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$= \dfrac{5\times0 + 100\times10}{5+10} = \dfrac{200}{3} = 66.66\,cm$

4. (b) We have three identical spheres, each of mass M



Using $X_{cm} = \dfrac{M_1X_1 + M_2X_2 + M_3X_3}{M_1 + M_2 + M_3}$

Similarly, $Y_{cm} = \dfrac{M_1Y_1 + M_2Y_2 + M_3Y_3}{M_1 + M_2 + M_3}$

$X_{cm} = \dfrac{M\times0 + M\times2 + M\times0}{3M} = \dfrac{2}{3}$

$Y_{cm} = \dfrac{M\times0 + M\times2 + M\times0}{3M} = \dfrac{2}{3}$

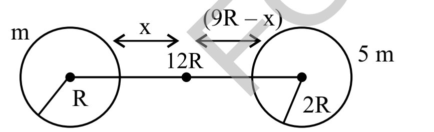
Position vector $= \dfrac{2}{3}\hat{i} + \dfrac{2}{3}\hat{j} = \dfrac{2}{3}\left(\hat{i} + \hat{j}\right)$

5. (d) Centre of mass may lie on centre of gravity net torque of gravitational pull is zero about centre of mass.

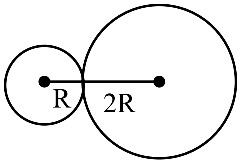
Mechanical advantage $= \dfrac{\text{Load}}{\text{Effort}} > 1$

Load > Effort

6. (b) Here, we have two spheres of mass m and 5m



Initial distance between their centers = 12R



At time of collision the distance between their centers = 3R

So total distance travelled by both = 12R – 3R = 9R

Since the bodies move under mutual forces, center of mass will remain stationary so

$m_1x_1 = m_2x_2 \Rightarrow mx = 5m(9R - x)$

$x = 45R - 5x \Rightarrow 6x = 45R$

$x = \dfrac{45}{6}R = 7.5R$

7. (a) Angular Speed (ω):

ω_1(second hand) $= \dfrac{2\pi}{60}$ rad/s (completes 2π in 60s).

ω_2 (minute hand) $= \dfrac{2\pi}{3600}$ rad/s (completes 2π in 3600s).

ω_3 (hour hand) $= \dfrac{2\pi}{43200}$ rad/s (completes 2π in 43200s).

Angular Distance in 1 Minute (x):

$x = \omega \times 60$ (since 1 minute = 60s).

$x_1 = 2\pi$ rad (second hand completes full circle).

$x_2 = \dfrac{2\pi}{60}$ rad (minute hand moves $\dfrac{1}{60}$th of a circle).

$x_3 = \dfrac{2\pi}{720}$ rad (hour hand moves $\dfrac{1}{720}$th of a circle).

For the second hand:

$\dfrac{\omega_1}{x_1} = \dfrac{\frac{2\pi}{60}}{2\pi} = \dfrac{1}{60}$

For the minute hand:

$\dfrac{\omega_2}{x_2} = \dfrac{\frac{2\pi}{3600}}{\frac{2\pi}{60}} = \dfrac{1}{60}$

For the hour hand:

$\dfrac{\omega_3}{x_3} = \dfrac{\frac{2\pi}{43200}}{\frac{2\pi}{720}} = \dfrac{1}{60}$

All ratios equal $\dfrac{1}{60}$ so $k = \dfrac{1}{60}$ is constant.

Hence, options 1 is correct.

8. (a) The angular acceleration direction is given along angular velocity or opposite to angular velocity depending upon whether angular velocity magnitude is increasing or decreasing and this direction remains along the axis of circular motion.

9. (c) $\omega = \omega_0 + \alpha t$

$\alpha = \dfrac{\omega - \omega_0}{t}$

where $\omega_0 = 1200$ rpm $= \dfrac{1200}{60}\times 2\pi$ rad/s

and $\omega = 3120$ rpm $= \dfrac{3120}{60}\times 2\pi$ rad/s

$\alpha = 4\pi$ rad/s^2

10. (d) Initial angular speed of wheel, $\omega_0 = 2\pi f_0$

$= 2\pi \times \dfrac{360}{60}$ rad/s $= 12\pi$ rad/s

Final angular speed of wheel, $\omega = 2\pi f$

$= 2\pi \times \dfrac{1200}{60}$ rad/s $= 40\pi$ rad/s.

Hence, t = 14 sec, Now, $\omega = \omega_0 + \alpha t$

$\Rightarrow a = \dfrac{\omega - \omega_0}{t} = \dfrac{40\pi - 12\pi}{14} = \dfrac{28\pi}{14}$

$= 2\pi$ rad/s^2

11. (a) Particle at periphery will have both radial and tangential acceleration

$a_t = R\alpha = 0.5 \times 2 = 1\ ms^{-2}$

Final angular speed, $\omega = \omega_0 + \alpha t$

$\omega = 0 + 2 \times 2 = 4$ rad/sec

$a_c = \omega^2 R = (4)^2 \times 0.5 = 16 \times 0.5 = 8\ ms^{-2}$

$$a_{total} = \sqrt{a_t^2 + a_c^2} = \sqrt{1^2 + 8^2} \approx 8\, ms^{-2}$$

12. (a) The position of point P on rod through which the axis should pass so that the work required to set the rod rotating with minimum angular velocity ω_0 is their center of mass so

$$m_1 x = m_2 (L - x) \Rightarrow x = \frac{m_2 L}{m_1 + m_2}$$

13. (d) Moment of inertia about XY

$I_{XY} = I_{CM} + mR^2$

For a solid sphere: $I_{CM} = \frac{2}{5} mR^2$

Thus:

$$I_{XY} = \frac{2}{5} mR^2 + mR^2 = \frac{7}{5} mR^2$$

But $I_{XY} = mk^2$:

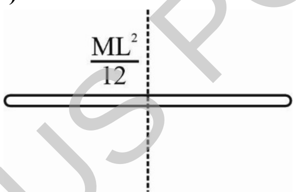
$$\frac{7}{5} mR^2 = m(5)^2$$

$$R^2 = \frac{25 \times 5}{7}$$

$$R = \frac{5\sqrt{5}}{\sqrt{7}} = \frac{5x}{\sqrt{7}}$$

Thus, $x = \sqrt{5}$.

14. (c)



$$\frac{ML^2}{12} = 2400 \Rightarrow \frac{400 L^2}{12} = 2400$$

$$\Rightarrow L^2 = \frac{2400 \times 12}{400} \Rightarrow L^2 = 72$$

$$\Rightarrow L = \sqrt{72} \Rightarrow L = 8.5\ cm$$

15. (None)

Radius of gyration of a solid surface,

$$K_S = \sqrt{\frac{2}{5}} R$$

Radius of gyration of a hollow surface,

$$K_H = \sqrt{\frac{2}{3}} R$$

$$\Rightarrow \frac{K_S}{K_H} = \sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}}$$

16. (b) Initial angular speed, ω_i = 60 rpm

$= 60 \times \frac{2\pi}{60} = 2\pi$ rad/s

Final angular speed ω_f = 360 rpm

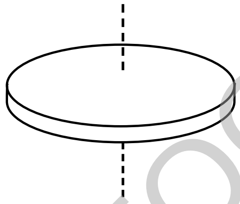
$= 360 \times \frac{2\pi}{60} = 12\pi$ rad/s

Energy spent $= \Delta kE = \frac{1}{2} I\ (\omega_f^2 - \omega_i^2)$

$$486 = \frac{1}{2} \times I \times [(12\pi)^2 - (2\pi)^2]$$

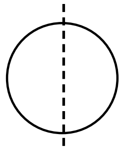
$$I = \frac{2 \times 486}{140\pi^2} \approx 0.7\ \text{kg-m}^2$$

17. (c) Moment of inertia of the disc about an axis perpendicular to the disc of passing through the centre.



$$I_1 = \frac{mR^2}{2}$$

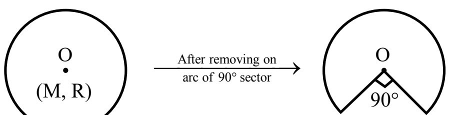
Moment of inertia of the disc about an axis in the plane of the disc and passing through the centre.



$$I_2 = \frac{mR^2}{4}$$

$$\Rightarrow \frac{k_1}{k_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{mR^2/2}{mR^2/4}} = \sqrt{2} : 1$$

18. (d)



Mass of new ring, $M' = M - \frac{M}{4} = \frac{3M}{4}$

Radius, R

Required moment of Inertia, $I_R = M'R^2$

$$\Rightarrow I_R = \left(\frac{3M}{4}\right) R^2 \Rightarrow I_R = \frac{3}{4} MR^2$$

So, value of 'K' is $\frac{3}{4}$.

Hence, correct option is (d).

19. (c) From work energy theorem, to stop the system

$$W_{ext} = \frac{1}{2} I\omega^2 \Rightarrow W_{ext} \propto I$$

$$I_C > I_B > I_A \Rightarrow W_C > W_B > W_A$$

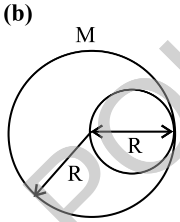
20. (d) $K.E_{rotation} = \frac{1}{2} I\omega^2$

$$E_{sphere} = \frac{1}{2} I_s \omega^2 = \frac{1}{2} \times \frac{2}{5} MR^2 \times \omega^2$$

$$E_{cylinder} = \frac{1}{2} I_c (2\omega)^2 = \frac{1}{2} \times \frac{MR^2}{2} \times 4\omega^2$$

$$\frac{E_{sphere}}{E_{cylinder}} = \frac{1}{5}$$

21. (b)



Mass per unit area of disc $= M/\pi r^2$

Mass of removed portion of disc

$$= \frac{M}{\pi R^2} \times \pi \left(\frac{R}{2}\right)^2 = \frac{M}{4}$$

Moment of inertia of removed portion about an axis passing through centre of disc.

$$I'_0 = \frac{1}{2} \times \frac{M}{4} \times \left(\frac{R}{2}\right)^2 + \frac{M}{4} \times \left(\frac{R}{2}\right)^2$$

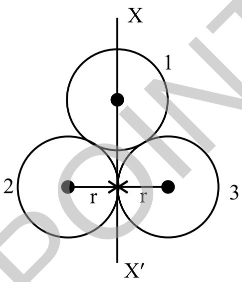
$$I'_0 = \frac{3MR^2}{32}$$

Moment of inertia of disc with removed portion is

$I = I_0 - I_0'$

$$I = \frac{1}{2} MR^2 - \frac{3MR^2}{32} \Rightarrow I = \frac{13\ MR^2}{32}$$

22. (c)



$$I_{xx'} = I_1 + I_2 + I_3$$

$$\frac{2}{3} mr^2 + \left(\frac{2}{3} mr^2 + mr^2\right) + \left(\frac{2}{3} mr^2 + mr^2\right)$$

(Using parallel axis theorem)

$\Rightarrow I_{xx'} = 2mr^2 + 2mr^2 = 4mr^2$

23. (a) The torque acting on the wheel,

$$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{1000}{300} = \frac{1}{3}\ \text{rad/sec}^2$$

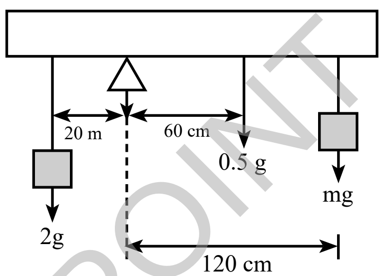
We have given that initial angular speed is zero, so

$\omega_i = 0$

$\omega_f = \omega_i + \alpha t = 0 + \frac{1}{3} \times 3$

$\omega_f = 1$ rad/sec

24. (c)



For balanced torque, net torque on the system will be zero

2g × 20 = 0.5 g × 60 + mg × 120 ⇒ 40 = 0.5 × 60 + 120 m

⇒ 40 – 30 = 120 m ⇒ 10 = 120 m

$\Rightarrow 1 = 12\text{ m} \Rightarrow m = \frac{1}{12}\text{ kg}$

25. (b) $\vec{F} = 3\hat{j}\text{N}, \vec{r} = 2\hat{k}$

$\vec{\tau} = \vec{r}\times\vec{F} = 2\hat{k}\times 3\hat{j} = 6(\hat{k}\times\hat{j}) = 6(-\hat{i})$

$\vec{\tau} = -6\hat{i}\text{ Nm}$

26. (a) angular displacement, $\theta = 2\pi$ revolution

$\theta = 2\pi \times 2\pi = 4\pi^2$rad

$\omega_i = 2\pi f = 2\pi\times\frac{3}{60}\text{rad/s}\left(\because f = \frac{3}{60}s^{-1}\right)$

$\omega_f = 0$

Work energy theorem.

$W = \frac{1}{2}I(\omega_f^2 - \omega_i^2) \Rightarrow -\tau\theta = \frac{1}{2}\times\frac{1}{2}mr^2(0^2 - \omega_i^2)$

$\Rightarrow -\tau = \frac{\frac{1}{2}\times\frac{1}{2}\times 2(4\times 10^{-2})^2\left(-3\times\frac{2\pi}{60}\right)^2}{4\pi^2}$

$\Rightarrow \tau = 2\times 10^{-6}\text{ Nm}$

27. (d) As $\tau_{ext} = 0$ Angular momentum will remain conserved.

28. (d) Torque,

$\vec{\tau} = \vec{r}\times\vec{F}$ [$\vec{r}$ is position vector and $\vec{F}$ is force]

$\vec{\tau} = (0\hat{i} + 2\hat{j} - \hat{k})\times(4\hat{i} + 5\hat{j} - 6\hat{k})$

$\vec{\tau} = -7\hat{i} - 4\hat{j} - 8\hat{k}$

29. (b) Hollow cylinder is equivalent to ring

$I_{ring} = MR^2$

$\tau = I\alpha$ … (*i*)

$\tau = FR$ … (*ii*)

$I\alpha = FR$

$MR^2\alpha = FR \Rightarrow MR\alpha = F$

$\alpha = \frac{F}{MR} \Rightarrow \frac{30}{3\times 40\times 10^{-2}} \Rightarrow \alpha = 25\text{ rad/s}^2$

30. (a)

$K_A = K_B \Rightarrow \frac{L_A^2}{2I_A} = \frac{L_B^2}{2I_B}$

As $I_B > I_A$ So, $L_A^2 < L_B^2 \Rightarrow L_A < L_B \Rightarrow L_B > L_A$

31. (b) Velocity of the automobile

$v = 54\times\frac{5}{18} = 15\text{ ms}^{-1}$

$\omega_0 = \frac{v}{R} = \frac{15}{0.45} = \frac{100}{3}\text{ rad/s}$

So angular acceleration

$\alpha = \frac{\Delta\omega}{t} = \frac{\omega_f - \omega_0}{t} = -\frac{100}{45}\text{ rad/s}^2$

$\tau = I\alpha = 3\times\frac{100}{45} = 6.66\text{ kgm}^2/\text{s}^2$

32. (b) Since, net torque is zero, so angular momentum is conserved

$\vec{\tau} = \vec{r}\times\vec{F} = 0\ (-12 + 12\alpha)\hat{j} + (6 + 6\alpha)\hat{k} = 0$

$\Rightarrow \alpha = -1$

33. (b) Angular momentum remains constant because of the torque of tension is zero.

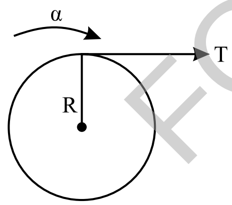
So, initial angular momentum = final angular momentum

$\Rightarrow L_i = L_f \Rightarrow mv_0R = mv\frac{R}{2}$

$\Rightarrow v = 2v_0$

$KE_f = \frac{1}{2}m(2v_0)^2 = 2mv_0^2$

34. (d)



Here, mass of the cylinder, M = 50 kg

Radius of the cylinder, R = 0.5 m

Angular acceleration, α = 2 rev/s²

= 2 × 2π rad s⁻² = 4π rad s⁻²

Torque τ = TR

Moment of inertia of the solid cylinder about its axis,

$I = \frac{1}{2}MR^2$

∴ Angular acceleration of the cylinder

$\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$

$T = \frac{MR\alpha}{2} = \frac{50\times 0.5\times 4\pi}{2} = 157\text{ N}$

35. (d) In case of rolling, the velocity of the point of contact with the surface is zero.

Also, the velocity of the point at the highest point of the rolling body is twice the velocity of COM of the body.

So, point P moves faster than point Q.

36. (a) Work needed = change in kinetic energy

Here, Final KE = 0

Initial KE = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2$

$\left(\because I = \frac{1}{2}mr^2 \text{ and } \omega = \frac{v}{r}\right)$

$= \frac{3}{4}\times 100\times(20\times 10^{-2})^2 = 3\text{J}$

work needed = 3 J

37. (b) $\frac{K_t}{K_t + K_r} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$

$\Rightarrow \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mR^2\left(\frac{v^2}{R^2}\right)} = \frac{5}{7}$

38. (a) External torque = zero

By conservation of angular momentum

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$I_1 = I_2 = I$

$\omega = \frac{\omega_1 + \omega_2}{2}$

$KE_1 = \frac{1}{2}I\omega_1^2 + \frac{1}{2}I\omega_2^2$

$KE_2 = \frac{1}{2}(2I)\omega^2$

$KE_2 = I\left[\frac{\omega_1 + \omega_2}{2}\right]^2$

$\Delta KE = \frac{1}{4}I[\omega_1 - \omega_2]^2$

39. (b) Time taken by the body to reach the bottom when it rolls down on an inclined plane, $t = \sqrt{\frac{2l\left(1 + \frac{K^2}{R^2}\right)}{8\sin\theta}}$

$\frac{t_d}{t_s} = \sqrt{\frac{1 + \frac{R^2}{2R^2}}{1 + \frac{2R^2}{5R^2}}} = \sqrt{\frac{3}{2}\times\frac{5}{7}} = \sqrt{\frac{15}{14}} \Rightarrow t_d > t_s$

[t_d is time taken by disk and t_s is time taken by sphere]

40. (a) For rolling motion without slipping on inclined plane

$a_1 = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}$ [θ is the angle of inclined plane with the ground]

And for slipping motion on inclined plane

$a_2 = g\sin\theta$

Required ratio $= \frac{a_1}{a_2} = \frac{1}{1 + \frac{K^2}{R^2}} = \frac{1}{1 + \frac{2}{5}} = \frac{5}{7}$

41. (a) From conservation of mechanical energy

$= \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = mgh$

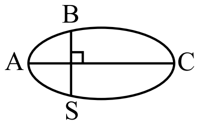
$\Rightarrow \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = mg\left(\frac{3v^2}{4g}\right) \Rightarrow \frac{K^2}{R^2} = \frac{1}{2}$

∴ The object is a Disc

CHAPTER 7 Gravitation

Kelper's Laws and Dynamics of Planetary Motion

1. The kinetic energies of a planet in an elliptical orbit about the Sun at positions A, B and C are K_A, K_B and K_C respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then *(2018)*



a. $K_B < K_A < K_C$ b. $K_A > K_B > K_C$
c. $K_A < K_B < K_C$ d. $K_B > K_A > K_C$

2. Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet, i.e., $T^2 = Kr^3$ here K is constant. If the masses of sun and planet are M and m respectively then as per Newton's law of gravitation force of attraction between them is $F = \frac{GMm}{r^2}$ here G is gravitational constant. The relation between G and K is described as: *(2015)*

a. $GMK = 4\pi^2$ b. $K = G$
c. $K = \frac{1}{G}$ d. $GM = 4\pi^2$

3. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio is *(2011 Mains)*

a. $\left(\frac{r_1}{r_2}\right)^2$ b. $\frac{r_2}{r_1}$ c. $\left(\frac{r_2}{r_1}\right)^2$ d. $\frac{r_1}{r_2}$

Newton's Law of Gravitation & Acceleration Due to Gravity

4. The mass of a planet is $\frac{1}{10}$th that of the earth and its diameter is half that of the earth. The acceleration due to gravity on that planet is: *(2024)*

a. 4.9 m s^{-2} b. 3.92 m s^{-2} c. 19.6 m s^{-2} d. 9.8 m s^{-2}

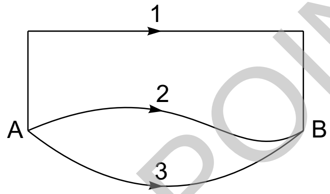
5. The mechanical quantity, which has dimensions reciprocal of mass (M^{-1}) is: *(2023-Manipur)*

a. angular momentum
b. coefficient of thermal conductivity
c. torque
d. gravitational constant

6. If R is the radius of the earth and g is the acceleration due to gravity on the earth surface. Then the mean density of the earth will be *(2023-Manipur)*

a. $\frac{\pi RG}{12g}$ b. $\frac{3\pi R}{4gG}$ c. $\frac{3g}{4\pi RG}$ d. $\frac{4\pi G}{3gR}$

7. A gravitational field is present in a region and a mass is shifted from A to B through different paths as shown. If W_1, W_2 and W_3 represent the work done by the gravitational force along the respective paths, then: *(2022 Re)*



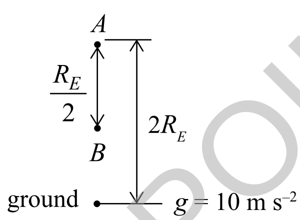
a. $W_1 < W_2 < W_3$ b. $W_1 = W_2 = W_3$
c. $W_1 > W_2 > W_3$ d. $W_1 > W_3 > W_2$

8. If the mass of the Sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which of the following is not correct? *(2018)*

a. Time period of a simple pendulum on the Earth would decrease
b. Walking on the ground would become more difficult
c. Raindrops will fall faster
d. 'g' on the Earth will not change

Variation in g Due to Altitude, Depth and other Factors

9. An object of mass 100 kg falls from point *A* to *B* as shown in figure. The change in its weight, corrected to the nearest integer is (R_E is the radius of the earth) *(2024 Re)*



a. 49 N b. 89 N c. 5 N d. 10 N

10. A body weighs 72 N on the surface of the earth. What is the gravitation force on it, at a height equal to half the radius of the earth? *(2020)*

a. 32 N b. 30 N c. 24 N d. 48 N

11. What is the depth at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value that at the surface of earth? (radius of earth = R) *(2020-Covid)*

a. $\frac{R(n-1)}{n}$ b. $\frac{Rn}{(n-1)}$ c. $\frac{R}{n}$ d. $\frac{R}{n^2}$

12. A body weighs 200 N on the surface of the earth. How much will it weigh half way down to the centre of the earth? *(2019)*

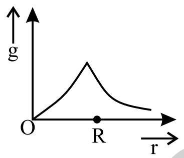
a. 150 N b. 200 N c. 250 N d. 100 N

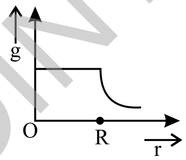
13. The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then: *(2017-Delhi)*

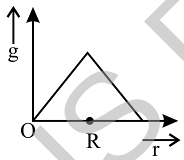
a. d = 1 km b. $d = \frac{3}{2}$ km

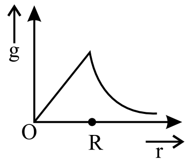
c. d = 2 km d. $d = \frac{1}{2}$ km

14. Starting from the center of the earth having radius R, the variation of g (acceleration due to gravity) is shown by: *(2016 - II)*

a. 

b. 

c. 

d. 

Gravitational Intensity, Potential and Potential Energy

15. Two bodies of mass m and 9m are placed at a distance R. The gravitational potential on the line joining the bodies where the gravitational field equals zero, will be (G = gravitational constant): *(2023)*

a. $-\frac{20Gm}{R}$ b. $-\frac{8Gm}{R}$ c. $-\frac{12Gm}{R}$ d. $-\frac{16Gm}{R}$

16. In a gravitational field, the gravitational potential is given by,

$$V = -\frac{K}{x}(J/Kg)$$

The gravitational field intensity at point (2, 0, 3)m is: *(2022)*

a. $+\frac{K}{4}$ b. $+\frac{K}{2}$ c. $-\frac{K}{2}$ d. $-\frac{K}{4}$

17. A body of mass 60 g experiences a gravitational force of 3.0 N, when placed at a particular point. The magnitude of the gravitational field intensity at that point is: *(2022)*

a. 180 N/kg b. 0.05 N/kg

c. 50 N/kg d. 20 N/kg

18. Match List-I and List-II *(2022)*

	List-I		List-II
A.	Gravitational constant (G)	(i)	$[L^2T^{-2}]$
B.	Gravitational potential Energy	(ii)	$[M^{-1}L^3T^{-2}]$
C.	Gravitational potential	(iii)	$[LT^{-2}]$
D.	Gravitational Intensity	(iv)	$[ML^2T^{-2}]$

Choose the correct answer from the options given below:

a. A-(iv), B-(ii), C-(i), D-(iii)
b. A-(ii), B-(i), C-(iv), D-(iii)
c. A-(ii), B-(iv), C-(i), D-(iii)
d. A-(ii), B-(iv), C-(iii), D-(i)

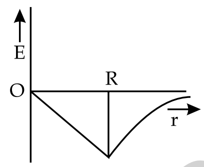
19. The work done to raise a mass m from the surface of the earth to a height h, which is equal to the radius of the earth, is: *(2019)*

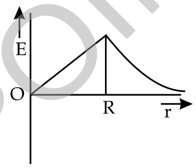
a. mgR b. 2mgR c. $\frac{1}{2}$mgR d. $\frac{3}{2}$mgR

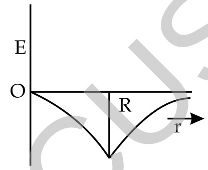
20. At what height from the surface of earth the gravitation potential and the value of g are -5.4×10^7 J kg^{-1} and 6.0 ms^{-2} respectively. Take the radius of earth as 6400 km: *(2016 - I)*

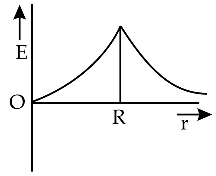
a. 2600 km b. 1600 km c. 1400 km d. 2000 km

21. Dependence of intensity of gravitational field (E) of earth with distance (r) from center of earth is correctly represented by: *(2014)*

a. 

b. 

c. 

d. 

22. Infinite number of bodies, each of mass 2 kg are situated on x-axis at distances 1 m, 2 m, 4 m, 8m, ... respectively, from the origin. The resulting gravitational potential due to this system at the origin will be: *(2013)*

a. –4G b. –G c. $-\frac{8}{3}G$ d. $-\frac{4}{3}G$

23. A body of mass 'm' taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change in potential energy of body will be: *(2013)*

a. $\frac{1}{3}$mgR b. 2 mgR c. $\frac{2}{3}$mgR d. 3 mgR

Satellite, Orbital Velocity and Escape Velocity

24. The escape velocity for earth is v. A planet having 9 times mass that of earth and radius, 16 times that of earth, has the escape velocity of: *(2024 Re)*

a. $\frac{V}{3}$ b. $\frac{2v}{3}$ c. $\frac{3v}{4}$ d. $\frac{9v}{4}$

25. The minimum energy required to launch a satellite of mass m from the surface of earth of mass M and radius R in a circular orbit at an altitude of 2R from the surface of the earth is: *(2024)*

a. $\frac{GmM}{2R}$ b. $\frac{GmM}{3R}$

c. $\frac{5GmM}{6R}$ d. $\frac{2GmM}{3R}$

26. A satellite is orbiting just above the surface of the earth with period T. If d is the density of the earth and G is the universal constant of gravitation, the quantity $\frac{3\pi}{Gd}$ represents: *(2023)*

a. $\sqrt{T}$ b. T c. T^2 d. T^3

27. The escape velocity of a body on the earth surface is 11.2 km/s. If the same body is projected upward with velocity 22.4 km/s, the velocity of this body at infinite distance from the center of the earth will be: *(2023-Manipur)*

a. $11.2\sqrt{2}$ km/s b. Zero

c. 11.2 km/s d. $11.2\sqrt{3}$ km/s

28. The escape velocity from the Earth's surface is v. The escape velocity from the surface of another planet having a radius, four times that of Earth and same mass density is: *(2021)*

a. 2v b. 3v c. 4v d. v

29. A particle of mass 'm' is projected with a velocity $v = kV_e$ $(k < 1)$ from the surface of the earth. The maximum height above the surface reached by the particle is: *(2021)*

a. $R\left(\frac{k}{1+k}\right)^2$ b. $\frac{R^2k}{1+k}$

c. $\frac{Rk^2}{1-k^2}$ d. $R\left(\frac{k}{1-k}\right)^2$

30. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is: *(2016 - I)*

a. 1 : 2 b. $1:2\sqrt{2}$ c. 1 : 4 d. 1: 2

31. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface, is: *(2016 - II)*

a. $\frac{2mg_0R^2}{R+h}$ b. $-\frac{2mg_0R^2}{R+h}$

c. $\frac{mg_0R^2}{2(R+h)}$ d. $-\frac{mg_0R^2}{2(R+h)}$

32. A remote-sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and $g = 9.8$ m/s^2, then the orbital speed of the satellite is: *(2015 Re)*

a. 6.67 km/s b. 7.76 km/s

c. 8.56 km/s d. 9.13 km/s

33. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then: *(2015 Re)*

a. The acceleration of S is always directed towards the center of the earth.

b. The angular momentum of S about the center of the earth changes in direction, but its magnitude remains constant.

c. The total mechanical energy of S varies periodically with time.

d. The linear momentum of S remains constant in magnitude.

34. A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass = 5.98×10^{24} kg) have to be compressed to be a black hole? *(2014)*

a. 10^{-2} m b. 10^{-6} m c. 10 m d. 100 m

Weightlessness

35. Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will: *(2017-Delhi)*

a. Move towards each other

b. Move away from each other

c. Will become stationary

d. Keep floating at the same distance between them

Answer Key

1. (b)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (a)	**10.** (a)
11. (a)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (d)	**16.** (d)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (a)
21. (a)	**22.** (a)	**23.** (c)	**24.** (c)	**25.** (c)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (b)
31. (d)	**32.** (b)	**33.** (a)	**34.** (a)	**35.** (a)					

Explanations

1. (b) When a planet revolves around the sun its angular momentum is conserved

$\Rightarrow$ mvr = constant

$\therefore \quad v \propto \frac{1}{r}$ $\quad$ [$\because$ mass is constant]

As $r_A < r_B < r_C$

$\Rightarrow v_A > v_B > v_C$

hence

$K_A > K_B > K_C$

2. (a) $T_p^2 = Kr^3$ given $\quad \ldots (i)$

As the planet revolves in nearly circular orbit

$$\text{Time Period} = \frac{\text{Circumference of the orbit}}{\text{Orbital speed}}$$

$$T_{planet} = \frac{2\pi r}{V_p} \quad \left[\because \text{orbital speed } V_p = \sqrt{\frac{GM}{r}}\right]$$

$$= \frac{2\pi r . r^{1/2}}{\sqrt{GM}}$$

$$T_p = \frac{2\pi r^{3/2}}{\sqrt{GM}} \quad \ldots(ii)$$

Squaring Eq. (*ii*), we have

$$T_p^2 = \frac{4\pi^2 r^3}{GM}$$

$$Kr^3 = \frac{4\pi^2 r^3}{GM}$$

$$\Rightarrow K = \frac{4\pi^2}{GM}$$

3. (b) According to the law of conservation of angular momentum

$L_1 = L_2$

$mv_1r_1 = mv_2r_2 \Rightarrow v_1r_1 = v_2r_2$

$$\frac{v_1}{v_2} = \frac{r_2}{r_1}$$

4. (b) $g = \frac{GM}{R^2} = 9.8 \text{ m/s}^2 \quad \ldots (i)$

Now for Planet,

$M' = \frac{M}{10}, D' = \frac{D}{2}$ [D is diameter of earth]

$\therefore \; R' = \frac{R}{2}$

Now, $g' = \frac{GM'}{(R')^2} = \frac{GM \times 4}{10 \times R^2} = \frac{4}{10} \times \frac{GM}{R^2}$

Substitute value of $\frac{GM}{R^2}$ from equation (*i*)

$g' = 0.4 \times 9.8 \text{ m/s}^2 = 3.92 \text{ m/s}^2$

5. (d) Gravitational constant = $[M^{-1}L^3T^{-2}]$

So, gravitational constant has power of (–1) of M.

6. (c) Acceleration due to gravity on the earth surface,

$$g = \frac{4}{3}\pi GR\rho, \; r = \frac{3g}{4\pi GR}$$

7. (b) Gravitational force is a conservative force work done by a conservative force does not depend on the path.

Therefore, $W_1 = W_2 = W_3$

8. (d) $\because \; g = \frac{GM}{R^2}$

If universal constant becomes

10 times, then

G' = 10G

New acceleration due to gravity is g'

$$= \frac{G'M}{R^2}, \; g' = \frac{10\,GM}{R^2} \quad \Rightarrow \quad g' = 10\,g$$

So g on earth surface will change

9. (a) Given:

Mass m = 100 kg,

At point A: Height $h = 2R_E$,

At point B: Height $h = \frac{3R_E}{2}$,

$g = 10 \text{ m/s}^2$.

Gravitational acceleration at height *h*:

$$g' = g\left(\frac{R_E}{R_E + h}\right)^2$$

At A ($h = 2R_E$):

$$g_A = \frac{g}{9}$$

At B $\left(h = \frac{3R_E}{2}\right)$:

$$g_B = g\left(\frac{R_E}{R_E + \frac{3R_E}{2}}\right)^2 = \frac{4g}{25}$$

Change in weight:

$$\Delta W = m(g_B - g_A) = 100\left(\frac{4 \times 10}{25} - \frac{10}{9}\right)$$

$= 49$ N

10. (a) Initial weight on the surface of earth

$W_1 = mg = 72N$

weight of the body at a height $h = \frac{R}{2}$

$$W_2 = mg' = \frac{mg}{\left(1+\frac{h}{R}\right)^2} = \frac{72N}{\left(1+\frac{R/2}{R}\right)^2} = \frac{72}{9/4}$$

$$\left[\because g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}\right]$$

$W_2 = 32N$

11. (a) Inside the earth at depth d from the surface

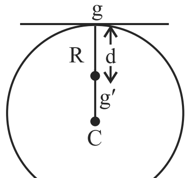
$$g_{eff} = g\left(1 - \frac{d}{R}\right)$$

As the new acceleration due to gravity becomes $\frac{1}{n}$ times,

$$\therefore g_{eff} = \frac{g}{n} \;\Rightarrow \frac{g}{n} = g\left(1 - \frac{d}{R}\right)$$

$$\Rightarrow d = \frac{(n-1)R}{n}$$

12. (d)



Acceleration due to gravity at a depth d from surface of earth

$$g' = g\left(1 - \frac{d}{R}\right) \quad \ldots (i)$$

Here g = acceleration due to gravity at earth's surface

Multiplying by mass 'm' on both sides of a.

$$mg' = mg\left(1 - \frac{d}{R}\right); \quad \left(\because d = \frac{R}{2}\right)$$

$$= 200\left(1 - \frac{R}{2R}\right) = \frac{200}{2} = 100\,N$$

13. (c) Acceleration due to gravity at a height 1 km above earth's surface

$$a_h = g\left[1 - \frac{2h}{R}\right] = g\left[1 - \frac{2 \times 1}{R}\right] = g\left[1 - \frac{2}{R}\right]$$

Acceleration due to gravity at depth 'd' below earth surface

$$a_d = g\left[1 - \frac{d}{R}\right]$$

$\because \; a_h = a_d$

$$\Rightarrow g\left[1 - \frac{2}{R}\right] = g\left[1 - \frac{d}{R}\right]$$

d = 2 km

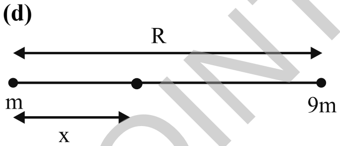
14. (d) For $0 < r \le R_e$

$$g = \left(\frac{GM_e}{R_e^3}\right)r \Rightarrow g \propto r$$

For $r > R_e$

$$g = \frac{GM_e}{r^2} \Rightarrow g \propto \frac{1}{r^2}$$

15. (d)



Let the gravitational field is zero at a distance x from the mass m.

$$\frac{Gm}{x^2} = \frac{G9m}{(R-x)^2}$$

$\Rightarrow R - x = 3x$ or $x = \frac{R}{4}$

Gravitational potential at $\frac{R}{4}$

$= -\frac{Gm}{\frac{R}{4}} - \frac{G9m}{\frac{3R}{4}}$

$= -\frac{4Gm}{R} - \frac{12Gm}{R} = -\frac{16Gm}{R}$

16. (d) The gravitational potential,

$v(x) = -\frac{K}{x}$

$E_g = -\frac{dv}{dx} = -\frac{d}{dx}\left(\frac{-k}{x}\right)$

$\vec{E}_g = -\frac{K}{x^2}\hat{i}$

Now $|\vec{E}_g(2, 0, 3)| = \frac{-K}{(2)^2} = \frac{-K}{4}$

17. (c) Gravitational field intensity is

$I_g = \frac{F}{m} = \frac{3}{60\times10^{-3}} = 50\text{ N/kg}$

[$\because$ m = 60g = 60×10^{-3}kg]

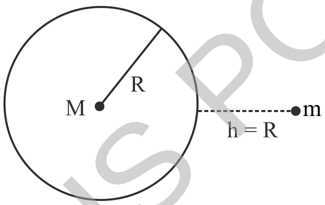
18. (c) Gravitational constant = $[M^{-1}L^3T^{-2}]$

Gravitational potential energy = $[ML^2T^{-2}]$

Gravitational potential = $[L^2T^{-2}]$

Gravitational intensity = $[LT^{-2}]$

19. (c)



Initial gravitational potential energy

$U_i = \frac{-GMm}{R}$

Final gravitational potential energy at height h = R

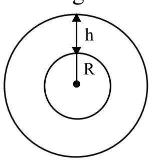
$U_f = \frac{-GMm}{(R+R)}$

As work done = Change in PE

$\therefore W = U_f - U_i$

$= \frac{GMm}{2R} = \frac{gR^2m}{2R} = \frac{mgR}{2} \quad (\because GM = gR^2)$

20. (a) Magnitude of Gravitational Potential at a height h



$= \frac{GM}{R+h} = 5.4\times10^7\text{ JKg}^{-1}$ … (*i*)

$g = \frac{GM}{(R+h)^2} = 6\text{ m/s}^2$ … (*ii*)

Dividing eqn. (*i*) by (*ii*)

$(R+h) = \frac{5.4\times10^7}{6} = 0.9\times10^7\text{ m} = 9000\text{ km}$

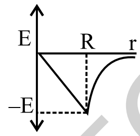
h = 9000 km – 6400 km = 2600 km

21. (a) Gravitational field intensity

For a point inside the earth $E = \frac{-GMr}{R^3}$

For a point outside the earth $E = \frac{-GM}{r^2}$

Where –ve sign indicates that the gravitational field is attractive



Accurate graph to show variation of E with r

22. (a) Total Gravitation Potential

$= V = \frac{-GM}{r_1} + \left(\frac{-GM}{r_2}\right) + ...$

$\Rightarrow V = -G(2)\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right]$

Because it forms geometric progression and for a geometric progression

$S_{GP} = \frac{r^n - 1}{r - 1}$, where $r = \frac{1}{2}$

$S_{GP} = \frac{\left(\frac{1}{2}\right)^n - 1}{-1/2} = \frac{-1}{-1/2} = 2$

$\therefore V = -2G \times S_{GP}$

$= -2G \times 2 = -4G$

23. (c) Change in gravitational P.E. = $U_f - U_i$

$= -\frac{GMm}{3R} - \left(-\frac{GMm}{R}\right)$

$= \frac{2}{3}\frac{GMm}{R} = \frac{2}{3}mgR$

24. (c) Given:

Escape velocity for Earth = *v*.

For the planet:

Mass (M_p) = $9M_e$ (where M_e is Earth's mass),

Radius (R_p) = 16 R_e (where R_e is Earth's radius).

Escape velocity,

$v_e = \sqrt{\frac{2GM}{R}}$

Therefore,

$\frac{(v_e)_p}{(v_e)_e} = \sqrt{\frac{M_p}{R_p} \times \frac{R_e}{M_e}}$

$\frac{(v_e)_p}{v} = \sqrt{\frac{9M_e}{16R_e} \times \frac{R_e}{M_e}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

$(v_e)_p = \frac{3}{4}v$

25. (c) $(T.E)_{initial}$ = PE of satellite

$= \frac{-GMm}{R}$

$(T.E)_{final}$ = P.E of satellite + K.E of satellite

$= \frac{-GMm}{3R} + \frac{1}{2}m\left(\sqrt{\frac{GM}{3R}}\right)^2 = \frac{-GMm}{6R}$

Minimum energy required

$= \frac{-GMm}{6R} + \frac{GMm}{R}$

$= \frac{-GMm + 6GMm}{6R} = \frac{5GMm}{6R}$

26. (c) Time period of satellite

$T = 2\pi\sqrt{\frac{R^3}{GM}}$

$= 2\pi\sqrt{\frac{R^3}{Gd\frac{4}{3}\pi R^3}} \Rightarrow T = \sqrt{\frac{3\pi}{Gd}}$

$\Rightarrow \frac{3\pi}{Gd} = T^2$

27. (d) $V_\infty = \sqrt{V^2 - V_e^2}$

We have given that,

$V = 2V_e$

So, By using the above formula,

$V_\infty = \sqrt{(2V_e)^2 - V_e^2}$

$V_\infty = \sqrt{3}V_e = 11.2\sqrt{3}$ km/s

28. (c) Escape velocity from the earth's surface is,

$V = \sqrt{\frac{2GM}{R}} \Rightarrow V = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho}$

$\left[\begin{array}{l}\because \text{Mass} = \text{Volume} \times \text{density} \\ \therefore M = \frac{4}{3}\pi R^3 \times \rho\end{array}\right]$

$\Rightarrow V \propto R$

$\Rightarrow \frac{V}{V_e} = \frac{R}{4R} \Rightarrow V_e = 4V$

29. (c) Given,

$v = Kv_e$

where,

v_e = escape velocity = $\sqrt{\frac{2GM}{R}}$

Let maximum height reached above the surface of earth be 'h'.

Total energy remains conserved at each point.

$\therefore$ Total energy at the surface of earth = Total energy at height h

$\Rightarrow \frac{1}{2}mv^2 - \frac{GmM}{R} = 0 - \frac{GmM}{(R+h)}$

[$\because$ at max^{m} height, velocity becomes zero.]

$\Rightarrow \frac{1}{2}mv^2 - \frac{GmM}{R} = -\frac{GmM}{(R+h)}$

$\Rightarrow \frac{1}{2}m(KV_e)^2 = -\frac{GmM}{(R+h)} + \frac{GmM}{R}$

$$\Rightarrow \frac{1}{2}K^2\left(\sqrt{\frac{2GM}{R}}\right)^2 = GM\left[\frac{R+h-R}{R(R+h)}\right]$$

$$\Rightarrow \frac{1}{2}K^2\left(\frac{2GM}{R}\right) = GM\left[\frac{h}{R(R+h)}\right]$$

$$\Rightarrow K^2 = \frac{h}{(R+h)} \Rightarrow RK^2 + hK^2 = h$$

$$\Rightarrow h = \frac{RK^2}{1-K^2}$$

30. (b) $V_e = \sqrt{2gR}$

$\because \; g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 d,$

Where d is density

$$\Rightarrow \; g = \frac{4}{3}\pi GdR$$

$$\Rightarrow \; V_e = \sqrt{2 \times \frac{4}{3}\pi GdR.R} = \sqrt{\frac{8}{3}\pi GdR^2}$$

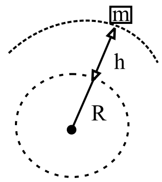
$$V_p = \sqrt{\frac{8}{3}\pi G\, 2d\,(2R)^2} \quad [\because d_p = 2d, R_p = 2R]$$

$$\frac{V_e}{V_p} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

31. (d) PE of the satellite at a height h $= \frac{-GMm}{R+h}$

$$KE = \frac{1}{2}m\left[\sqrt{\frac{GM}{R+h}}\right]^2$$

$$\left[\because \text{Orbital velocity} = \sqrt{\frac{GM}{R+h}}\right]$$



$$KE = \frac{GMm}{2(R+h)}$$

$$TE = \frac{-GMm}{R+h} + \frac{GMm}{2[R+h]}$$

$$= \frac{-GMm}{2(R+h)}\frac{R^2}{R^2} \quad \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{-g_0 mR^2}{2[R+h]}$$

32. (b) For the satellite revolving around earth

$$v_0 = \sqrt{\frac{GM_e}{(R_e+h)}} = \sqrt{\frac{GM_e}{R_e\left(1+\frac{h}{R_e}\right)}} = \sqrt{\frac{gR_e}{1+\frac{h}{R_e}}}$$

Substituting the values

$v_0 = \sqrt{60 \times 10^6}$ m/s

$v_0 = 7.76 \times 10^3$ m/s

$= 7.76$ km/s

33. (a) Acceleration due to earth to the satellite is centripetal, hence directed towards centre.

Angular momentum conservation holds good for comparable masses but $M_{earth} >> M_{satellite}$

34. (a) Escape velocity $(v_e) = \sqrt{\frac{2GM}{R}} =$ c = speed of light

$$\Rightarrow R = \frac{2GM}{c^2} = \frac{2 \times 6.6 \times 10^{-11} \times 5.98 \times 10^{24}}{(3 \times 10^8)^2} m$$

$= 10^{-2}$ m

35. (a) Since two astronauts are floating in gravitational free space, the only force acting on the two astronauts is the gravitational pull of their masses, $F = \frac{Gm_1m_2}{r^2}$ which is attractive in nature.

Hence they move towards each other.